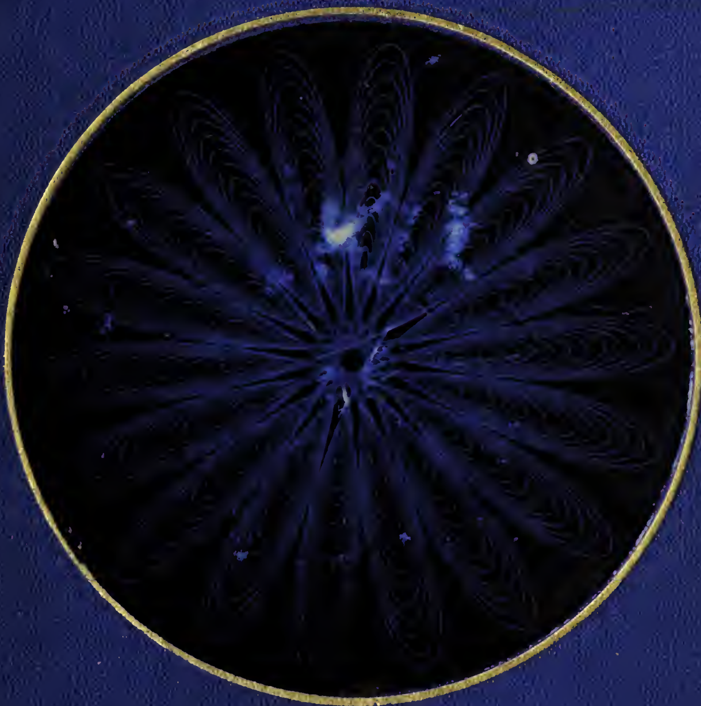
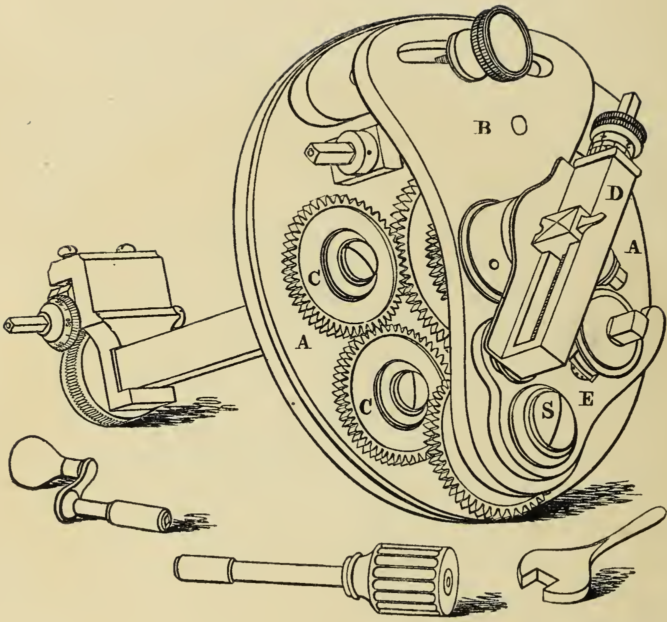


Epicycloidal Cutting Frame

LONDON: PRINTED BY
SPOTTISWOODE AND CO., NEW-STREET SQUARE
AND PARLIAMENT STREET



Frontispiece.

NOTES ON THE

EPICYCLOIDAL CUTTING FRAME

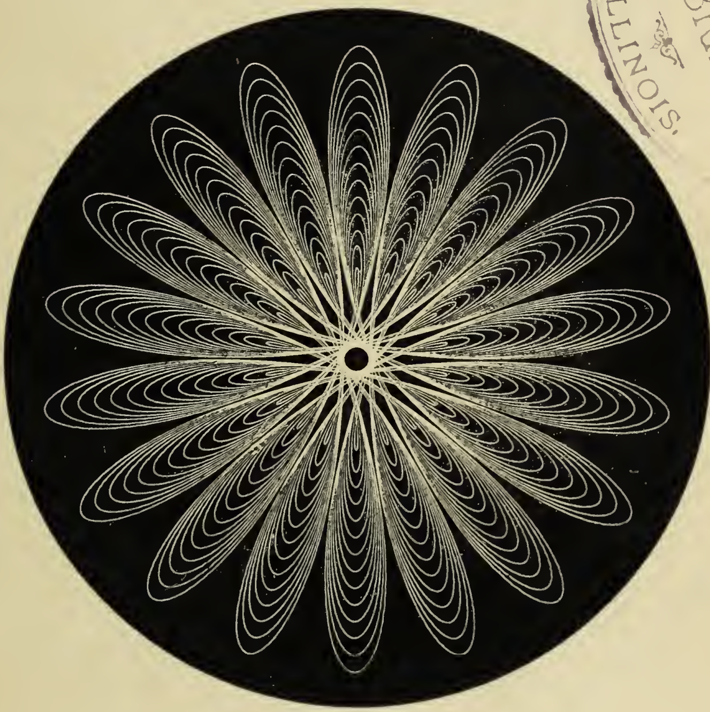
OF

MESSRS. HOLTZAPFFEL & CO.

WITH SPECIAL REFERENCE TO ITS COMPENSATION ADJUSTMENT

AND WITH

NUMEROUS ILLUSTRATIONS OF ITS CAPABILITIES



BY THOMAS SEBASTIAN BAZLEY M.A.

LONDON

TRÜBNER AND CO., 60 PATERNOSTER ROW

1872

PREFACE.

WHEN an amateur of the Art of Turning adds to his apparatus the Geometric Chuck in one or more parts—or the Elliptical or Epicycloidal Cutting Frame—he will probably, if at all prone to scientific dabblings, soon desire to know something of the laws upon which the action of such instruments depends, and will not be contented with admiring the facility with which, by haphazard adjustments, this species of ornamentation can be applied to the decoration of plane surfaces. After puzzling his own brains for a while, it will perhaps occur to him that some enthusiastic predecessor may have placed on record in a simple and attractive form the result of such an investigation as he is disposed to commence. But here he is doomed to disappointment; for the following list comprises, so far as can be ascertained, all that has been hitherto published upon the subject.

1. "Geometrical and Graphical Essays." Adams: London, 1791. One of these is a description, with drawings, of the "geometric pen," invented by J. Baptist Suardi, to whose work upon the instrument some reference is made, depending upon "the com-

pound motion of two circles, one moving round the other." The description is concise, and is accompanied by a few simple examples, adjustments for which are given in the forms of "radius ratio," "velocity ratio," and "direction."

2. "Manuel du Tourneur." L. E. Bergeron: Paris, 1816. 2nd edition. This fine old work, which may sometimes be met with, is a record of the Art of Turning as it existed half a century ago, upon a scale which, for the present period, has not been equalled, nor even attempted, with the exception of the three volumes of the late Mr. Holtzapffel's important but unfinished work. At vol. ii., page 326, is a full account, with engravings and specimens, of a "machine épicycloïde," resembling what is now known as the Geometric Chuck. There is no reference to the theory of the subject, beyond a statement of the general principles by the aid of which "on peut boucler ou ne pas boucler." The examples are apparently designed with a view to inlaying the curve with strips of tortoiseshell, and do not include the fine engraving which forms so attractive a feature of Geometric turning as now practised.

3. In the *Mechanics' Magazine*, Old Series, there appeared, in 1829 and subsequently, some lively correspondence on the compensatory division of the periphery of an ellipse, in which incidental mention was made, by Child, Ibbetson, and others, of the "Geometric Chuck" and its performance.

4. "A Brief Account of Ibbetson's Geometric Chuck." London, 1833. This account is "brief" enough, and equally unsatisfactory, being composed in great measure of a continuation of the controversy in the *Mechanics' Magazine.* The pamphlet contains some pleasing examples of simple geometric turning, but all information as to their origin, and as to the construction of the chuck itself, is withheld. Mr. Ibbetson states that he invented his chuck without having seen or heard of the "Manuel du Tourneur," and considers that its capabilities, as arranged by himself, largely exceed those of Suardi's pen.

5. In "Ibbetson's Circular Turning," London, n. d., by the same author, there is some explanation of the manner in which "a line passed through the centres of the circles," which constitute his examples of compound excentric turning, "will form the path of a true epicycloidal curve."

6. "The Art of Double Counting." By Captain Ash. London, 1857. This work entirely supersedes the last-named, and contains many ingenious and original designs, with ample instructions for their execution. Isolated portions of epicycloidal curves, especially their looped extremities, traced by the imaginary centres of circles or ellipses, form a special feature of Captain Ash's method.

7. "Treatise on Mathematical Drawing Instruments." By Stanley. London, 1866. Among the instruments of Mr. Stanley's manufacture is an im-

proved geometric pen, which, from the examples given on page 88, certainly appears to possess some unusual facilities for variation in single curves. Tables of settings are furnished for the attainment of a few of the principal effects.

8. "Lathes and Turning," by C. H. Northcott (London, 1868), has a special chapter on the Geometric Chuck, with an engraving of its single form and examples of much merit, produced by the double or "two-part" chuck. By way of explaining the principles of epicycloidal motion, 39 "laws" are enunciated; but their language, though doubtless strictly accurate, is so formidably scientific, that the amateur is much to be pitied who, with no more elementary treatise than this, and with no previous knowledge of the subject, seeks to obtain an insight into the principles by which these simple and compound curves are traced.

9. "The Lathe and its Uses." Trübner, 1869. A capital book, and, though discursive, and not fortunate in engravings, likely to be of much assistance to the rising amateur generation. At its conclusion is an engraving of the Compound Geometric Chuck as constructed by Mr. Plant, with an outline description and a few examples, but no real explanation. Bergeron's chapter upon the "machine épicycloïde" is also introduced in part.

10. A volume published at Philadelphia in 1869, by "E. J. W., Lennox, Mass.," contains thirty photographs, from patterns traced on blackened card, and

two pages of letterpress. Many of the designs possess much elegance and originality, and some of them appear to have been effected by the Elliptical Cutting Frame (invented by Captain Ash), to which extra wheels have been added, enabling that instrument to produce 3 and 5 "consecutive" loops, of the external form only, besides the 2 (ellipse) and 4, to which it was at first restricted. There is no example of "circulating" curves.

11. In the *English Mechanic* of December 4, 1870, is a description, by Mr. Plant, more general than that given in "The Lathe and its Uses," of the Geometric Chuck and its necessary change-wheels.

12. "Patterns for Turning," by H. W. Elphinstone (Murray, 1872). A maximum of result with a minimum of apparatus. Mr. Elphinstone's method depends upon the principle that any point whatever, in a surface attached to the Lathe mandrel, may, by the horizontal movement of the Slide-rest combined with the circular movement of the Division plate, be brought into line with the axis of the Eccentric Cutting Frame. Curves of all kinds, when referred to polar co-ordinates, may thus be traced by a series of dots or circles; but the preliminary calculations, however interesting, must involve no small amount of labour.

Other Treatises upon the Art of Turning have been published, but without any notice of this branch; and one of the best, though a short one and not recent, is in "Rees's Cyclopædia," with an engraving and de-

scription of one of Holtzapffel and Deyerlein's lathes and accompanying chucks. There is no mention, however, of any contrivance for the production of epicycloidal curves; unless the Rose Engine may to some extent be so considered.

The illustrated description of the "Elliptical Cutting Frame," which has been appended, for about twenty years, to Messrs. Holtzapffel's Catalogue, will probably be known to the generality of readers into whose hands the present work may fall; and, with some modifications, is perfectly applicable to the instrument which we are about to consider.

If, after consulting with much interest, but without much benefit, some of the foregoing works, the amateur is fortunate enough to try the "Penny Cyclopædia," Knight & Co., 1843, he will find in vol. xxv., under the heading of "Trochoidal Curves," almost all the theoretical information he can desire. But, unless his mathematical attainments are considerable, he will not find it easy to follow all the steps of the reasoning, nor to interpret the theory as applied to the mechanism which he may have before him. No attempt is made in the following pages (Appendix excepted) to treat the subject "mathematically;" and complaint may not unreasonably be made of the too frequent exclusion of the negative sign, which should indicate a change in the direction of motion, and which has been omitted as much as possible with the view of facilitating the practical application of the formulæ. The more "popular"

portion of the article just named is transcribed with such expansion as seemed necessary; and its theoretical principles are applied, as there developed, to the explanation of the Epicycloidal Cutting Frame.

The author hopes that those fellow-amateurs who remember a little Trigonometry, and have not quite forgotten their Algebra, may find these "Notes" intelligible; and that, even if the explanatory portion be considered gratuitous, its practical result as regards the rules for "compensation" may not be unwelcome.

Although it is hardly possible to put the Epicycloidal Cutting Frame in motion, whatever be its adjustments, without obtaining a figure of symmetry and pleasing appearance, it is to be understood that the diagrams accompanying the letterpress are not proposed as subjects worthy of imitation, excepting, perhaps, figs. 65, 67, 87, and a few of the concluding examples. They are offered solely in illustration of the functions of the instrument, and not as favourable specimens of its performance. The amateur will find more satisfaction in executing his own designs than in becoming a mere copyist; and with the formulæ for correcting the angular deviation of the curves produced, which it is the object of this work to make known, he will be able to maintain the symmetry of the figure, however complicated.

THOS SEBASTIAN BAZLEY.

HATHEROP: *June*, 1872.

CONTENTS.

CHAPTER		PAGE
I.	Descriptive and Theoretical	1
II.	Development of Curves with "Consecutive" Loops	21
III.	Development of Curves with "Circulating" Loops	38
IV.	Investigation of Formulæ for Correcting the Obliquity due to the Radial Action of the Flange	53
V.	Examples of this Correction	75
VI.	Interpolation of Curves and their Spiral Arrangement	94
VII.	Hints on Design and on Treatment of the Instrument	106

CHAPTER PAGE

VIII. EXTENSION OF THE INSTRUMENT BY THE INTRODUCTION OF A SECOND PAIR OF CHANGE WHEELS . 130

IX. METHOD OF OBTAINING CONSECUTIVE EXTERNAL LOOPS OF HIGH NUMBERS 154

X. CURVES WITH CONSECUTIVE INTERNAL LOOPS OF HIGH NUMBERS SIMILARLY PRODUCED.—RECIPROCAL CIRCULATING CURVES 171

APPENDIX 189

Errata

Page 3,	line 22,	*insert* and inde-	*at the end of line*		
,, 19,	,, 22,	*for* is		*read* is	
,, 25,	,, 9,	,,	$n = 5$	,,	$n = -5$
,, 89,	,, 23 and 26,	,,	C	,,	**C**
,, 108,	,, 20,	,,	$(a \quad b)$	,,	$(a + b)$
,, 155,	,, 13,	,,	fig. 101	,,	fig. 102
,, ,,	,, 14,	,,	fig. 102	,,	fig. 101
,, 179,	,, 8,	,,	52, and 52,	,,	50 and 52
,, 182,	,, 21,	,,	$\frac{36}{42}$	,,	$\frac{36}{42} \times \frac{30}{60}$

NOTES

ON THE

EPICYCLOIDAL CUTTING FRAME

OF

MESSRS. HOLTZAPFFEL & CO.

CHAPTER I.

DESCRIPTIVE AND THEORETICAL.

WHEN two or more independent circular movements, in parallel planes, are combined so that their separate excentricities and angular velocities shall have an united effect upon the path of the point which renders visible this aggregate motion, the result is one of the many varieties of *trochoidal curves.*

The point may be fixed while the surface which receives the delineation of the path revolves by the agency of the combined circular movements; or the surface may be stationary while the point revolves under guidance of a similar character. The former is the system upon which the Geometric Chuck is constructed; and the latter is the principle of the "geometric pen," and of the Elliptical Cutting Frame, figured and described in the appendix to Messrs. Holtzapffel & Co.'s Catalogue of 1853. Of this instrument the Epicycloidal Cutting Frame, now to be

described, is an extension, designed by Mr. W. W. Pomeroy, the able superintendent of the Lathe and Tool Manufactory of Messrs. H. & Co.

The mechanism in its various parts, and their relation to one another, will be better understood by examination of the engraving which forms the frontispiece to these Notes, and which is introduced by the courteous permission of the makers.

The large driving pulley (A), having one groove only, rotates upon the enlarged cylindrical end of a spindle passing through the square stem of the instrument, which is placed as usual in the receptacle of the Slide Rest. This end of the spindle carries—in front of the pulley, and almost in contact with its surface—a wheel of 64 teeth, which remains absolutely fixed and motionless, except when the spindle itself is moved on its axis, by means of the tangent wheel of 96 teeth, and attached micrometer screw of 50 divisions, seen at its other extremity, for a purpose subsequently explained.

Immediately before the pulley, and parallel thereto, is placed the "Radial Flange" (B), fitting by a socket at one side upon the stud (S), near the circumference of the pulley, with which the opposite and wider side of the Flange nearly coincides. Between them, however, and bolted to the pulley, is interposed a stout bar, whose upper edge is formed into a portion of the circle which the Flange describes,—moving upon the stud as centre;—and is engraved with a scale of lines read by a single mark upon the adjoining edge of the Flange. The space, thus intervening between the Pulley and Flange, is required to accommodate other toothed wheels:—the stud upon which the Flange socket moves as a centre carrying two, whereof one (32) is always connected with the central (64), and the other

(60), upon the same stud as an axis, drives the remainder of the train. The contact of the (32) with the (64) is not direct, but made by either one, or two, "carrier"* wheels (C), one or both of which can be employed at pleasure. The axes of these "carriers" have a small range of adjustment, enabling them to be secured, by binding screws at the back of the pulley, in the necessary positions, whether in or out of action.

The radial Flange, moving upon the stud (S) as a centre, is actuated by a thin steel screw, attached, with some freedom of self-adjustment in angular direction, to the face of the pulley, and passing through a pin projecting internally from the Flange. A milled-headed screw clamps the Flange to the pulley by pressing on the parallel edges of a curved mortise in the former concentric with the stud.

In front of the Flange, as part of the same casting, and in such a position as to be central with the axis of the instrument when no radial excentricity is given to the Flange, is a cylindrical socket, receiving the axis, or spindle, of an "Eccentric Frame" (D), similar in all respects to that which, with a longer spindle pendent pulley, constitutes the well-known "Eccentric Cutting Frame." Upon the hinder end of this axis, and rotating always with it, is firmly screwed a wheel of (40) teeth. This (40) wheel is connected with the

* "Carrier." This word is not met with in treatises upon wheel work, but is commonly employed by many practical mechanics to denote one or more wheels, of any numbers of teeth, whose office is to *carry forward* the motion from one axis to another without affecting the velocity of the train. Professor Willis, and others who have followed his standard work on the Principles of Mechanism, use the expression "*idle wheel*" (which is rather hard upon a wheel that has, equally with the rest, to transmit the whole power employed, except what may be lost by friction). The terms "*intermediate*" and "*connecting wheel*" have also been adopted, but do not convey the intended meaning more accurately than the shorter word "carrier."

(60) on the stud (S) by two change wheels placed upon a removable arbor carried by the radial steel plate (E), seen in front of the instrument. One square-headed binding screw serves to fix the arbor in a mortise of the plate, and another, almost concealed in the engraving by the Eccentric Frame (D), secures the plate upon the Flange. Sufficient range of adjustment in these respects is provided, that whatever may be the change wheels upon the arbor—*and no change wheels are employed elsewhere*—they can be made to gear smoothly with the (60) and (40) wheels between which they are placed.

There are twelve change wheels supplied with the instrument, of 30, 32, 34, 36, 38, 40, 42, 44, 46, 48 (two), and 60 teeth respectively; which, besides the ellipse and straight line, give figures with 2, 3, 4, 5, or 6 "consecutive" loops, inwards, or outwards; and many others with from 7 to 90 "circulating" loops; some of which are hardly distinguishable from those containing equal loops of direct formation, and requiring more complicated apparatus.

The movement of the "Flange" upon the stud brings the axis of the "Frame" into a condition of excentricity as regards the axis of the instrument. From the centre of the stud to that of the Frame axis is precisely 2 inches; and from the same centre to the opposite edge of the Flange is 4·5 inches. Careful and repeated measurements confirm these figures, certainly to within 0·01 inch. The scale graduated upon the edge of the Flange contains 100 divisions, and (taken as the chord) measures 2·25 inches. And since the radius of the circular arc described by the axis of the Frame is to that of the arc described, from the same centre, by the edge of the

Flange, as 2 to 4·5 (i.e. as 1 : 2·25), it follows that the chords of their corresponding arcs will be in the same proportion. Consequently 2·25 inches on the Flange are equivalent to a radius of 1 inch for the imaginary circle described by the Frame axis; and the subdivisions of the Flange are arranged in the same proportion; not equally, but in a diminishing series, in the ratio of a scale of chords. In other words, one division upon the engraved edge of the Flange denotes, (at whatever part of the graduations it be taken), an excentricity of 0·01 inch in the position of the axis of the Eccentric Frame which it carries. The screw which carries the little tool-box on the Frame (D) is as usual of a multiple thread, whose effect is equivalent to that of a screw of ten threads to the inch; and its milled head micrometer marks divisions of half-hundredths, and less by estimation. It is very desirable that the Frame itself should also be graduated to tenths of an inch; which can be easily read by the circular edge of the collar of the binding screw. The facility thus given for determining, by inspection only, the excentricity of the tool upon the Frame, saves much wear and tear of the screw, and diminishes the risk of error in its adjustment.

When the driving pulley is caused to rotate, the Flange accompanies it, maintaining the degree of excentricity which it may have received; and the various wheels are carried round at the same time with a motion of rotation upon their respective axes derived from contact successively transmitted from the (64) fixed central wheel. The result is, that the Frame revolves, either in the same direction with the Flange, or opposed to it, according to the disposition of the "carriers," and with an angular velocity bearing a

certain ratio to that of the pulley : to which, it will be remembered, the Flange is fixed. If, during this combined action, the point of the tool which may have been placed in the Eccentric Frame remains central (i.e. is situated in the axis of that Frame), and no excentricity be given to the Flange, the tool makes simply a dot. If excentricity be given either to Flange or Frame, while the other remains central, the result is a circle whose radius is equal to that excentricity. If, however, both Flange and Frame be placed excentrically, certain curves will be traced, by the point of the tool, depending as to their size and characteristics upon the extent of the excentricities imparted, the value of the train of wheels employed, and the identity, or otherwise, of the directions in which the Flange and Frame are moving.

In order to have some control over the angular direction of the axis of such curves as this instrument is calculated to produce, a constant point of reference, or "*initial position*," is assumed ; to which it is in most cases desirable that the instrument should be brought before definitely fixing the change wheels, and before giving any excentricity to the Flange. The position adopted is when the Flange and Frame are at right angles to one another, and the Frame is at the same time, also perpendicular to the lathe bearers. The latter adjustment is easily obtained with the assistance of a " square " ; and in order to determine the horizontality of the Flange, a line is marked by the makers upon the edge of the cylindrical termination of the steel stem of the instrument, while another line is marked upon the edge of the narrow gun-metal cylinder which forms the back of the pulley. These short cylinders are of the same diameter, and the latter

rotates in contact with the former. If, when these two lines are coincident, and the Flange and micrometer screw of the tangent wheel are both at zero, the change wheels can be brought satisfactorily into gear without disturbing the verticality of the Frame,—the axis of the curve (for which the instrument may be afterwards adjusted) will be placed vertically, after the tabulated correction * has been made, at the tangent wheel, for the excentricity of the Radial Flange. It is probable, however, that the wheels will not gear in these exact positions; and if so, the tangent wheel must be moved by its screw until the Frame remains vertical, and the two lines coincide; the change wheels as well as the "carriers" having been already placed suitably. The reading of the micrometer screw, after this adjustment has been accomplished, will become the zero point of the tangent wheel, so long as the change wheels and the "carriers" remain undisturbed. It is absolutely essential that the Flange shall have no excentricity whatever, while this preliminary detail is being settled.

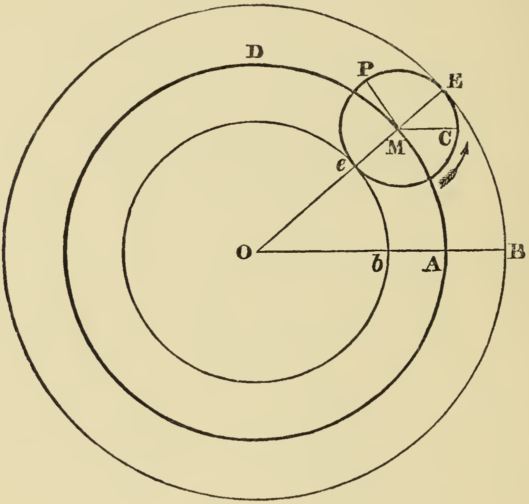
To offer any intelligible explanation of the performance of the "Epicycloidal Cutting Frame," it will be necessary to enter somewhat fully into the principles upon which such contrivances depend: and it is impossible to treat the subject more completely than has already been done in the article upon "Trochoidal Curves" in the "Penny Cyclopædia." What now follows is derived from the simpler portions of that treatise, and the same scientific nomenclature is preserved which is there adopted. Only such parts have been transcribed and amplified as seemed specially ap-

* The necessity for this "correction," and its extent in various cases, will be discussed subsequently.

plicable to the present investigation; but the reader is strongly recommended to refer to the Article in its integrity.

Let the point M (fig. 1) be carried uniformly round the circumference of the fixed circle AMD, whose centre is O, and radius OM = OA; and let M be the centre of another circle, PEC, whose radius MC = AB. This circle, PEC, is thus supposed to be carried by

Fig. 1.



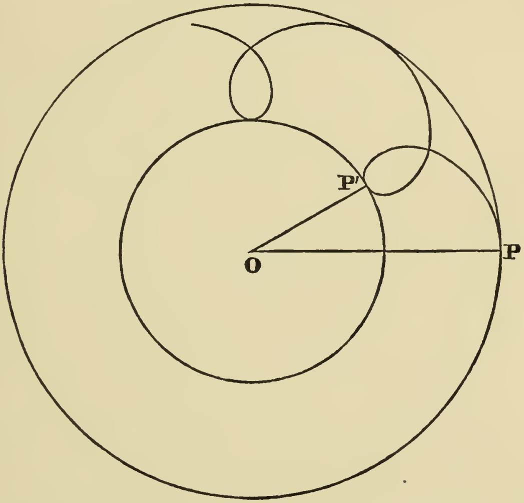
its centre around the circumference of the circle AMD, but *without rotating* upon its own axis, M.

Let the point P be carried uniformly round the circumference of the circle PEC (in the same manner as M is carried round that of AMD), so that its angular velocity from a line of fixed direction in the moving circle—such as MC, which is always parallel to AB—always bears a given proportion to the angular velocity of M:—say that of n to 1. That is, supposing P and M

to have started together, M from A, and P from B (M C being thus coincident with A B), the angles M O A and P M C would be described in equal times. But the latter would be n times greater than the former; or, P M C = n.M O A.

The point P, as it proceeds in its course, under the influence of this double movement, will trace out a curve which is called "trochoidal," or "planetary;"

Fig. 2.



and, as the circle P E C, on the circumference of which P is placed, is always contained between the two circles B E and *b e*, the curve marked out by P will also be bounded by these two circles. When P is at B, or any other part of the circle B E, it is at its greatest distance from the centre of that circle, or "in *apocentre*," and when at *b*, or at any other part of the circle *b e*, it is at its nearest point to the common centre of B E and *b e*, or "*in pericentre*."

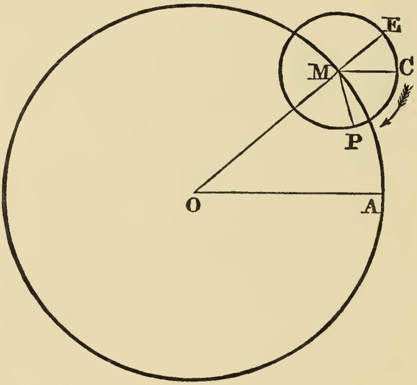
Let, therefore, these two circles B E, *b e*, be called "*apocentral*" and "*pericentral*" respectively.

Let the fixed circle A M D be called the "*deferent*," and the moving circle P E C be called the "*epicycle.*"

Let the angle M O A be called the "*deferential angle*," and the angle P M C, which is always $= n.$M O A, be called the "*epicyclic angle.*"

When the revolution of P is in the same direction (upwards from B) as that of M, let it be said that the "*epicycle is direct*;" and when in the contrary direction (downwards from B), let it be said that the "*epicycle is retrograde*," and let the direct motion be considered positive, having the sign $+$, and the retrograde motion negative, having the sign $-$.

Fig. 3.

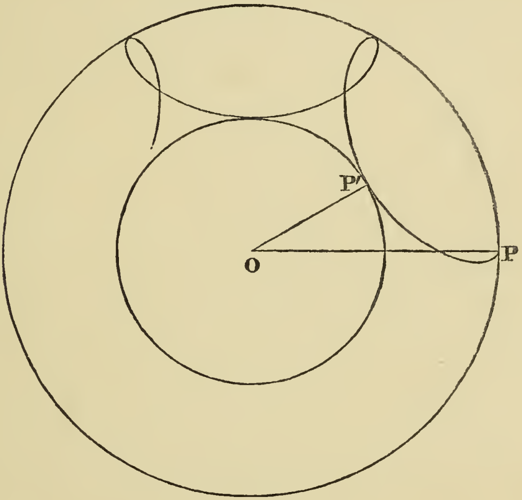


Let the radius of the deferent				A M D $= a$
,,	,,	,,	epicycle	P E C $= b$
∴	,,	,,	apocentral circle	B E $= a + b$
and	,,	,,	pericentral ,,	$b\,e = a - b$
let the deferential angle				M O A $= \phi$
∴ the epicyclic angle				P M C $= n\,\phi$

Also, let the motion of P, in the circumference of the epicycle, be not less than the motion of M in the circumference of the deferent; and let the motion of M in the deferent be always direct.

Now suppose M to be at A, and consequently P and C to be together at B; and suppose the epicycle to be direct. Then as P begins to move upwards, it will trace a curve of the kind shown in fig. 2, and there

Fig. 4.



will be a pericentre as soon as P M C has gained two right angles upon E M C or M O A; for by that time P will be at the circle *b e*. That is, a pericentre will occur when $n\,\phi - \phi = 180°$, i.e., when $\phi = \dfrac{180°}{n-1}$.

But if the epicycle be supposed to be "retrograde," then (commencing as before with M at A and P at B) there will be a pericentre as soon as E M C and P M C together make two right angles—that is, when $\phi + n\,\phi$

$= 180°$, i.e., when $\phi = \dfrac{180°}{n+1}$. Fig. 3 shows the direction of motion, and fig. 4 the class of curve which is produced when the epicycle is retrograde.

If n be represented by the fraction $\dfrac{p}{q}$, which is in its lowest terms, where p and q are integers, the curve will be found to return into itself when M has completed q revolutions. And if the epicycle be "direct," there will be $(p-q)$ or $(q-p)$ apocentres, and as many pericentres; but if the epicycle be "retrograde," the number both of apocentres and pericentres will be $p+q$. Of course when n is a whole number, $q = 1$, and one revolution of M will trace the whole curve.

Applying these general results to interpret the construction and performance of the Epicycloidal Cutting Frame, it will be perceived that the circle A M D (figs. 1 and 3) called the "deferent" corresponds to the imaginary fixed circle round which the axis of the Eccentric Frame revolves; and its radius O M $= a$ is equal to the excentricity which has been given to the Radial Flange. Similarly the circle P E C, called the "epicycle," is that which is described, if the Flange be central, by the point of the tool carried by the Eccentric Frame, and its radius P M $= b$ is equal to the excentricity which has been given to the tool-box on that Frame. Also, when both of the "carriers" are employed, the Flange and Frame will be seen to revolve in the same direction, and the "epicycle is direct;" but when one "carrier" only is in use, the Flange and Frame will be seen to move in opposite directions, and the "epicycle is retrograde."

Let V denote the value of the train of wheels which transmit an accelerated motion from the pulley to the

axis of the Eccentric Frame, V being obtained, as in all other combinations of toothed wheels or pulleys, by multiplying the numbers of all the "drivers" together for the numerator—and the numbers of all the "driven" together for the denominator—of a fraction which is to be expressed in its simplest form. In the present case the fixed (64) at the front end of the spindle is the first driver; then come the "carriers," whether one or two, but which do not affect the value of the train; then the (32) and (60) on the Flange axis; next the two change wheels, which we may designate by the letters x and y respectively; and, lastly, the (40) on the axis of the Eccentric Frame. The whole stand thus:—

$$\frac{64}{32} \times \frac{60}{y} \times \frac{x}{40} = \frac{2}{1} \times \frac{3}{y} \times \frac{x}{2} = \frac{3x}{y};$$

where x is the wheel which gears into the (40) on the Frame, and is the one *first placed* on the removable arbor; y being the other change wheel, which is placed upon the first (x), and fixed tightly with it upon the arbor by a milled edged nut with fine thread. The value of V, therefore, for any given change wheels is readily found from the equation $V = \frac{3x}{y}$.

It would probably be supposed at first sight that V may be substituted for n in the formulæ which have been just explained, and that the Frame would revolve V times, while the pulley moves round once, whether they had both the same direction or the contrary. But there is another movement besides. The wheel on the Frame, in addition to the number of times it is caused to turn on its axis by the train, is also carried round in a circle once for every rotation of the pulley; and the Frame has therefore to move round *once more* than

the value of V, or *once less*, according as the Flange and Frame are travelling in the same or in contrary directions.

This can easily be verified by experiment. Let the change-wheels $x = 60$, $y = 30$, be placed in the train; that is, let $V = 6$. And let the position which either extremity of the Eccentric Frame (say its milled head) occupies with respect to any part of the pulley—the Flange binding-screw for instance—be carefully noted. If the pulley be now moved round once by hand, the number of revolutions may be counted which are made by the Frame during that interval. It will then be observed that when both "carriers" are employed, so that Flange and Frame move together in the same direction, the latter revolves *seven* times instead of *six* before returning to the assigned position; while, if one "carrier" be excluded, the Frame, now moving in the opposite direction to that of the Flange, will arrive at its destination in *five* turns instead of *six*. But in *each* case there will be six coincidences between the milled head of the Frame and the binding screw of the Flange;—that is, the former will pass the latter six times during one rotation of the pulley, whether the directions of motion be identical or opposed.

To take another example, the wheels $x = 32$, $y = 48$, which make $\frac{x}{y} = \frac{2}{3}$ and therefore $V = 2$, will show that for every turn of the pulley, the Frame revolves *thrice* when the two are moving together, but only *once* when they are moving in opposition. And the same law must prevail, whatever be the numbers of teeth in the change wheels.

It is, therefore, obvious that when the epicycle is "direct," $n = V + 1$. When the epicycle is "retro-

grade," both n and V are negative; but the difference between the "synchronal absolute revolutions"* of Flange and Frame is irrespective of sign, and in this case $n = V - 1$. Also since $V = \frac{3x}{y}$, we have n always $= \frac{3x}{y} \pm 1$, a relation that will be found useful subsequently.

The varieties of which these curves are susceptible clearly depend upon the values which may be given to each of the quantities n and $\frac{a}{b}$:—of which a and b are always positive, while n may be either positive or negative, and has the most influence of the three in determining the character of the curve.

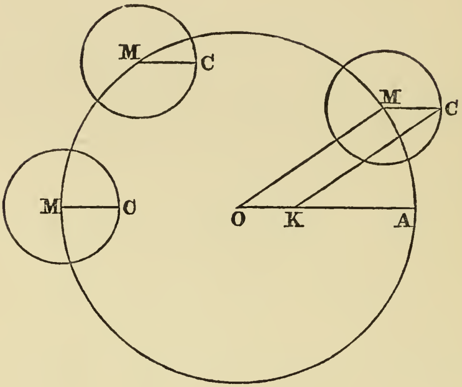
When $n = 1$, the epicyclic angle PMC (figs. 1 and 3) is always equal to the deferential angle MOA: therefore P is situated throughout the revolution, either at E or e, in the radius OM:—at E, if placed at B to begin with, and at e, if first starting from b. Consequently, the curve is here reduced to either the apocentral, or pericentral, circle, and possesses no practical interest.

When $n = 0$, the epicyclic angle PMC has no existence, and P makes no revolution on the circumference of the epicycle, but remains constantly at C. Now the line MC is carried round in a direction parallel to itself:—(for the epicycle is not supposed to possess any motion of rotation of its own:—it is P that revolves upon its circumference)—and C describes a circle equal to the deferent, but having its centre at K (fig. 5), OK being equal to the radius of the epicycle.

Neither of these cases ($n = 1$, or $n = 0$) applies to

* See *Principles of Mechanism*, by Professor Willis, 2nd edition, pp. 319-322. Longmans, 1870.

Fig. 5.



the instrument as now described, since by its mechanical construction V cannot be made sufficiently small. But, though it is not possible to give to the pulley so much greater an angular velocity than that which it must in consequence impart to the Eccentric Frame,* yet, when V is so far reduced as to be made equal to 2, —and the epicycle is retrograde,—n becomes $= - 1$. Under these conditions the Flange and Frame move in opposite directions with equal angular velocity, and the curve generated is an ellipse,—as may be shown thus.

In fig. 6, let the same letters of reference have the same signification as in figs. 1 and 3. The epicycle and the apocentral and pericentral circles are shown, but the deferent is omitted.

Upon the line O B, which has been taken as the axis of the curve, and to which M C is always parallel, let the perpendicular E N be drawn from E, a point in the apocentral circle.

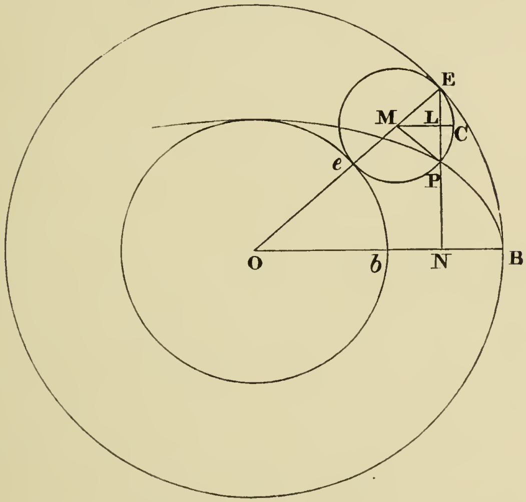
* There is an important exception to this, in an extended form of the instrument described in the last chapters.

Then, because M C is parallel to O N, the sides of the triangle E O N are cut proportionately (Euc. vi. 2).

$$\therefore \frac{EN}{EL} = \frac{EO}{EM}.$$

But, since the epicyclic angle C M P is, in this case, equal to the deferential angle M O N = E M L, it follows that the two triangles E M L, L M P are equal, and E P is bisected in L.

Fig. 6.



$$\therefore \frac{EN}{EP} = \frac{EO}{Ee}, \text{ and } \frac{EN}{PN} = \frac{EO}{Oe}.$$

Now E O $= a + b =$ half the major axis of the curve, and O $e = a - b =$ half its minor axis; for it has been shown that the curve, whatever be its form, will always be bounded by the apocentral and pericentral circles. And it is a property of the ellipse that the ordinate E N of the circle circumscribing the ellipse is to the corresponding ordinate P N of the ellipse as the major axis is to the minor axis: or that $\frac{EN}{PN}$ is constant.

Therefore in the figure before us EN : PN :: major axis : minor axis; and, as the same demonstration will apply wherever P be taken, it follows that when $n = -1$, P describes an ellipse.

The major axis of the ellipse produced is evidently $= 2(a + b)$, and the minor axis $= 2(a - b)$; from which it appears that a (= radius of deferent, = excentricity of Flange) = one-fourth of the sum of the two axes; and b (= radius of Epicycle, = excentricity of Frame) = one-fourth of their difference. The same rule applies to all curves obtained on this principle, and gives the means of calculating the adjustments of Flange and Frame, independently of the change wheels which it may be desired to use, in order that the curve, or pattern, shall occupy a specified extent of surface. Suppose, for instance, that it be desired to cover, more or less completely, an annular space whose exterior diameter is 0·8 inch, and its interior diameter 0·32 inch, i.e. that these are the diameters of the apocentral and pericentral circles respectively, within which the curve is to be placed.

The *sum* of the two given quantities is 112 hundredths of an inch; one-fourth of which is 28.

Their *difference* is 48; one-fourth of which is 12. Of these two dimensions 12 and 28, either may be appropriated at pleasure to a and the other to b. In the one case the loops will not pass beyond the centre, and in the other they will. Figs. 13 and 18 afford elementary examples of the two forms; and, if other change wheels be selected, the boundaries of the curve will continue the same, while a and b are undisturbed or are interchanged, however crowded may be the figure.

When $a = b$, the minor axis of the ellipse $= 2(b - b)$

= 0, or the ellipse becomes a straight line, whose length, = $2(a + a) = 4a$ = four times the excentricity on Flange, or on Frame; both those excentricities being now of the same amount.

In the case of the ellipse, it will be found immaterial, as regards the resulting curve, whether the value of (a) be transferred to the Flange, and of (b) to the Frame, or *vice versâ*; because the velocities of the two are equal. But this interchange cannot be made in the case of other curves, because (as is proved in the Treatise on *Trochoidal Curves* referred to), if (a) be taken for the radius of the epicycle, and (b) for that of the deferent, n must be replaced by $\frac{1}{n}$; or the Flange must go faster than the Frame; and for this ratio between their velocities the instrument as now described does not provide. This statement, however, requires some qualification; for there are three instances, standing at the head of the first column in Table III. (page 52), where it is possible, by using low numbers of teeth for (x) compared with those taken at the same time for (y), to have V less than 2. And under these circumstances, when the motion *is* inverse, the value of n, besides being negative, is less than 1, and the Flange does actually go faster than the Frame. The loops resulting from these combinations are all "circulating," and one of the three is rather fully illustrated in figs. 48 and 49.

The actual existence of the "deferent" and "epicycle," in connection with the Epicycloidal Cutting Frame, and their identity with the circles which have been shown to regulate the path of the curve, may be exemplified in the following manner. While the tool which is carried by the Frame is strictly central, and the Frame and

Flange are connected by the toothed wheels being placed in gear, let the Flange receive any convenient excentricity: the pulley being then rotated, a circle will be described by the point of the tool, concentric with the axis of the instrument, and whose radius is equal to the excentricity of the Flange. This circle is the *deferent.* The change wheels being now disconnected with the axis of the Frame, let the tool box on the latter be moved from its central position: let the pulley be now stationary, and the Frame axis be rotated separately: then the point of the tool will describe a circle whose centre is somewhere in the circumference of the former circle, and whose radius is equal to the excentricity of the Frame. This second circle is the *epicycle.*

Whatever be the change wheels now introduced, the curves which they yield, while the above excentricities remain unaltered, will all be situated within the annular space which is concentric with the first circle, and equal in width to the diameter of the second.

CHAPTER II.

DEVELOPMENT OF "CONSECUTIVE" CURVES.

THE distinction of curves as "consecutive" is here intended to imply that they are described by one revolution of the pulley; and that the loops, when formed, occur consecutively: it has no reference to the direction in which the epicycle may be moving. As a practical illustration, we will take an arrangement of wheels already suggested for a previous experiment, viz.: V = 6, ($x = 60$, $y = 30$), and observe the various phases of the curve which depend upon changes in the relative proportions of (a) the excentricity of the Flange, and (b) the excentricity of the tool box on the Frame. For the sake of symmetry ($a+b$) shall always be made equal to a fixed quantity, say 40 divisions of Flange or of Frame, i.e. four tenths of an inch. This will leave the invisible apocentral circle, within and in contact with which the curve is situated, always of the constant diameter 0·8 inch.

I. Employing in the first instance both "carriers," the loops, when formed, turn inwards; the epicycle is direct; and $n = 1 + V = 7$. Then,

1. If $a = 40$, $b = 0$, a circle only is produced; for there is no "epicycle," and the apocentral and pericentral circles both coincide with the "deferent." But,

2. If ever so small a value be given to b, the true circular outline is lost, as in fig. 7, where $a = 39·5$, $b = 0·5$. These figures (as in all other instances where

numerical values are affixed to a or b) denote hundredths of an inch, and therefore also the divisions to be taken at the Radial Flange, and the Eccentric Frame.

3. When $b = \frac{a}{n^2}$, the circle becoming further in-

Fig. 7.

Fig. 8.

flected, assumes somewhat of a rectilinear or polygonal form, as in fig. 8, where $a = 39{\cdot}2$, $b = 0{\cdot}8$. This is better seen in figs. 36 and 38, where V = 3 and 4 respectively, the loops being internal as in the present case.

Fig. 9.

Fig. 10.

4. Still increasing (b), and diminishing (a) *pro tanto*, so that ($a + b$) continues = 40,—the curve deviates yet further from a circle, and the six prominences and indentations become more developed: as in fig. 9, where $a = 38$, $b = 2$.

5. When (b) has increased so that $b = \frac{a}{n}$, the indentations become cusps—a distinct feature, whose attainment by this simple formula is more expeditious and satisfactory than by any method of trial.

Fig. 10, $a = 35$, $b = \frac{a}{n} = 5$.

Fig. 11.

Fig. 12.

6. Any further addition to the value of (b) now results in the production of loops, as in fig. 11, where $a = 33$, $b = 7$; and as $\frac{b}{a}$ increases, b being still less than a, the loops increase in size, and approach more

Fig. 13.

Fig. 14.

nearly both to one another and to the centre of the figure.

In fig. 12, $a = 31$, $b = 9$

" " 13, $a = 28$, $b = 12$.

7. When b = ?* the loops touch, as in fig. 14, where $a = 24$, $b = 16$.

Fig. 15.

Fig. 16.

8. Proceeding with the same kind of alteration in adjustment, the loops interlace, and approach the centre more nearly. Fig. 15, $a = 22$, $b = 18$.

9. And when $a = b$, the loops all pass through the centre, as in fig. 16, $a = b = 20$.

Fig. 17.

Fig. 18.

10. When (b) becomes greater than (a) the loops begin to overlap one another, and their extremities recede from the centre.

In fig. 17, $a = 16$, $b = 24$
" " 18, $a = 12$, $b = 28$.

11. They continue to enlarge considerably, and the

* See Appendix.

annular space which they occupy becomes more and more narrow.

Fig. 19, $a = 8, b = 32$
,, 20, $a = 3, b = 37$.

Fig. 19.

Fig. 20.

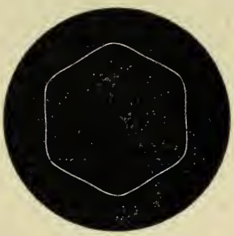
12. Till at length, when $(a) = 0$, the "deferent" vanishes, and the final circle is the "epicycle," coinciding with the apocentral and pericentral circles.

II. Let one "carrier" now be detached from the train; the loops, when they arise, turn outwards, the "epicycle" is retrograde, and $n = 5$.

1. When $a = 40$, $b = 0$, the result is, as in the former case, the "deferent" circle only.

Fig. 21.

Fig. 22.



2. When $a = 39{\cdot}5$, $b = 0{\cdot}5$, fig. 21, the interference with the circle is very marked; and,

3. When $b = \frac{a}{n^2} = 1{\cdot}6$, and $a = 38{\cdot}4$, fig. 22, the polygonal form is apparent, and more decided than in the corresponding fig. 8.

Fig. 23.

Fig. 24.

4. Increasing (b) and diminishing (a) as previously, we have in fig. 23, $a = 36$, $b = 4$.

5. And in fig. 24, where $b = \frac{a}{n} = 6{\cdot}7$ and $a = 33{\cdot}3$, the cusps, which are now inverted, attain their perfect termination. It is here to be observed that when, as

Fig. 25.

Fig. 26.

on this occasion, tenths of a division on Flange or Frame—i.e., thousandths of an inch—are specified in the adjustments, it is not to be inferred that such accuracy is attainable with certainty, or essential. The more nearly, however, such theoretical values may

happen to be translated into actual measurements, the more exact will be the results.

6. As (b) becomes gradually greater than $\frac{a}{n}$, the loops appear, increase, and approach, as in the corresponding figures obtained when n is positive.

Fig. 27.

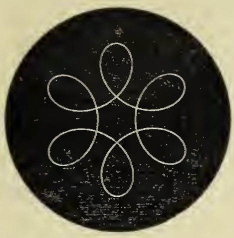


Fig. 28.

In fig. 25, $a = 31{\cdot}5$, $b = 8{\cdot}5$,
" " 26, $a = 28$ $b = 12$,
" " 27, $a = 25$ $b = 15$.

7. When b = ?* the loops touch, as in fig. 28, where $a = 22{\cdot}5$, $b = 17{\cdot}5$.

Fig. 29.

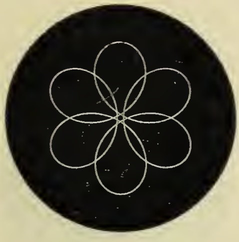


Fig. 30.

8. The loops then intersect and pass more nearly to the centre. Fig. 29, $a = 21$, $b = 19$.

9. And meet at the centre as formerly when $a = b = 20$. Fig. 30.

* See Appendix.

10. From this point there is a considerable resemblance to the figures in the previous case; the loops

Fig. 31.

Fig. 32.

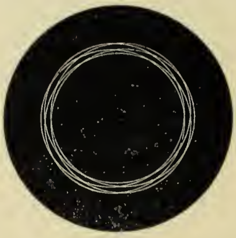
enlarge and recede as there shown, and the annular space contracts and vanishes, ultimately coinciding with the "epicycle," in the same manner.

In fig. 31, $a = 16$, $b = 24$,
" " 32, $a = 12$, $b = 28$,
" " 33, $a = 8$, $b = 32$,
" " 34, $a = 3$, $b = 37$.

III. It may be interesting to compare the development of similarly "consecutive" curves, (as distinguished from "circulating") for the other integral

Fig. 33.

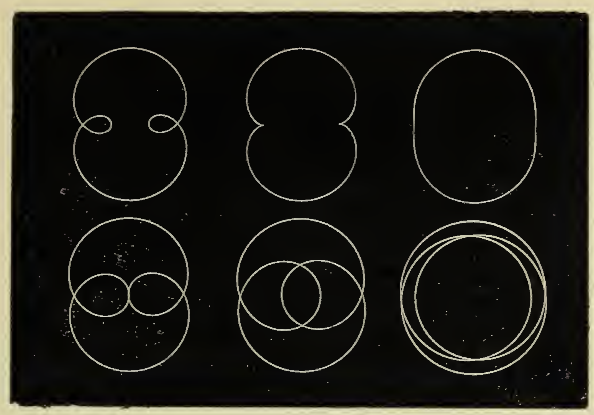
Fig. 34.



values of V, (2, 3, 4, and 5), of which the Epicycloidal Cutting Frame is susceptible. The more prominent features of each are given in the following diagrams,

which are upon the same scale as those preceding. The result of each adjustment, as stated below, will be readily recognised, without attaching a separate number to each figure.

Fig. 35.



1. ($x = 32, y = 48$, two carriers). V = 2, loops internal, $n = 3$, fig. 35.

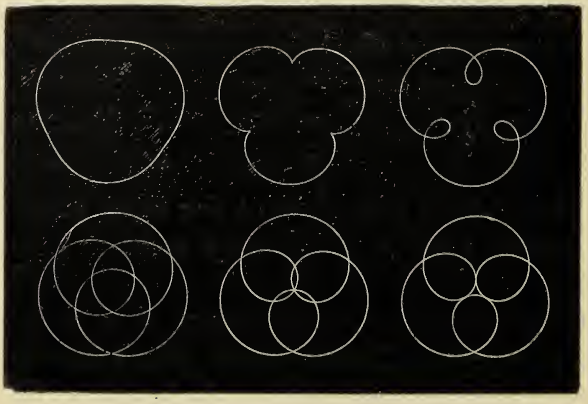
$a = 36,\ b = \frac{a}{n^2}$	$= 4,$	rectilinear.	
$a = 30,\ b = \frac{a}{n}$	$= 10,$	cusps.	
$a = 25,\quad b$	$= 15,$	loops.	
$a = b$	$= 20,$	,,	meet at centre.
$a = 15,\quad b$	$= 25,$	,,	intersect.
$a = 5,\quad b$	$= 35,$	,,	,,

As there are two loops only, the cases in which the loops touch, and in which they meet at centre, are identical.

2. ($x = 32, y = 48$, one carrier) V = 2, loops external, $n = -1$. The variety of two loops outwards does not exist, or rather is not included in the capa-

bilities of this instrument. The conditions which might be expected to produce this form, yield the ellipse in all proportions, as has been already explained. And the only change which can occur in the curve from its condition as an ellipse, except its return to a circle, is that of a straight line. For, when $n = -1$, the equations $b = \frac{a}{n^2}$, $b = \frac{a}{n}$, $b = a$, all mean the same thing *practically;* there can be no approximate rectilinear figure, no cusps, and no central intersection. With the change-wheels arranged as now stated, the instrument becomes an Elliptical Cutting Frame, and can be used for moderately heavy cuts, and for all such kinds of ornamentation as are the peculiar province of the latter instrument; and the method of angular correction, to which reference has already been made, and which will be discussed subsequently at greater length, is applied in the same manner as in the Elliptical Cutting Frame, and to the same extent, viz., equal divisions on Flange and at the Tangent-wheel micrometer.

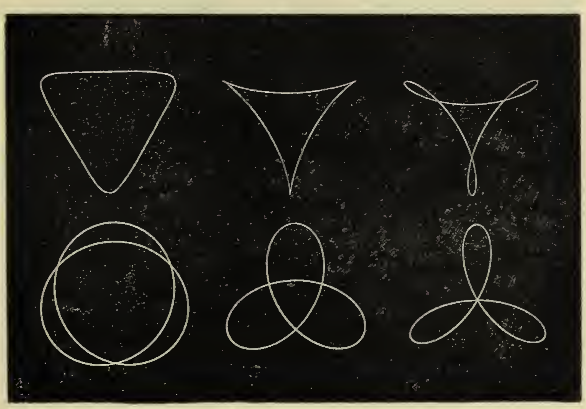
Fig. 36.



3. ($x = y = 48$, two carriers) V = 3, loops internal, $n = 4$, fig. 36.

$a = 37{\cdot}7$,	$b = \frac{a}{n^2} = 2{\cdot}3$,	rectilinear	
$a = 32$,	$b = \frac{a}{n} = 8$,	cusps	
$a = 27$,	$b = 13$,	loops	
$a = 20{\cdot}8$,	$b = 19{\cdot}2$,	,,	touch
$a = 20$,	$b = 20$,	,,	pass through centre
$a = 15$,	$b = 25$,	,,	intersect

Fig. 37.

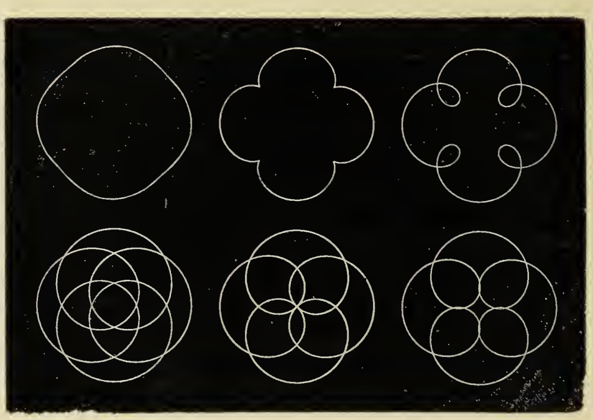


4. ($x = y = 48$, one carrier) V = 3, loops external, $n = -2$, fig. 37.

$a = 32$,	$b = \frac{a}{n^2} = 8$,	rectilinear	
$a = 26{\cdot}7$,	$b = \frac{a}{n} = 13{\cdot}3$,	cusps	
$a = 24$,	$b = 16$,	loops	
$a = 20$,	$b = 20$,	,,	meet at centre
$a = 15$,	$b = 25$,	,,	intersect
$a = 5$,	$b = 35$,	,,	,,

In this instance no side contact of loops is possible, as is evident from the course of the curve. It would hardly be supposed, from the appearance of the six figures in this diagram, fig. 37, that they are all, externally, of the same size; there is, for example, much apparent difference between the second and sixth. But the application of a pair of compasses will show that the radius of the circumscribing (i.e. the apocentral) circle is uniform throughout.

Fig. 38.

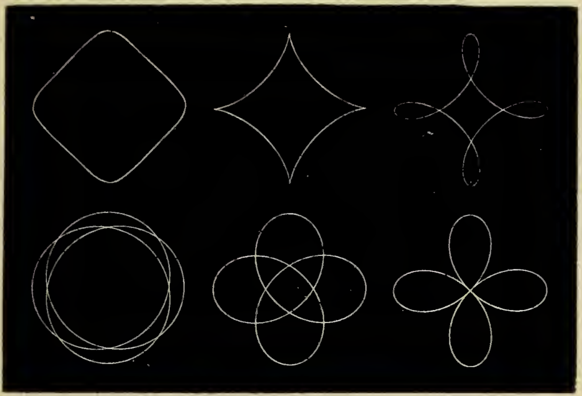


5. (x = 48, y = 36, two carriers). V = 4, loops internal, n = 5. Fig. 38.

$a = 38{\cdot}5$,	$b = \frac{a}{n^2} = 1{\cdot}5$,	rectilinear
$a = 33{\cdot}6$,	$b = \frac{a}{n} = 6{\cdot}4$,	cusps
$a = 28$,	$b = 12$,	loops
$a = 22$,	$b = 18$,	,, touch
$a = 20$,	$b = 20$,	,, pass through centre
$a = 15$,	$b = 25$,	,, intersect

6. ($x = 48$, $y = 36$, one carrier) V = 4, loops external, $n = -3$. Fig. 39.

Fig. 39.



$a = 36$, $b = \frac{a}{n^2} = 4$,	rectilinear	
$a = 30$, $b = \frac{a}{n} = 10$,	cusps	
$a = 25$, $b = 15$,	loops	
$a = 20$, $b = 20$,	,,	meet at centre
$a = 15$, $b = 25$,	,,	intersect
$a = 5$, $b = 35$,	,,	,,

The "rectilinear" figure in this case is the "square" which is employed in an exceedingly ornamental design among those published by Messrs. Holtzapffel & Co. in illustration of the Elliptical Cutting Frame, and which is also noticed by Captain Ash, the inventor of that instrument, at page 55, plate 13, of his work upon "Double Counting." * The directions for adjustment given by both writers are that the excentricities of Flange and Frame should be in the proportion of 8 : 1.

* London : Booth, Regent-street, and Holtzapffel & Co. 1857.

And their dimensions are $a = 40$, $b = 5$, and $a = 24$, $b = 3$; differing a little from the formula $b = \frac{a}{n^2}$, which would give 4·4 instead of 5 for the value of b when a is taken at 40; and 2·6 instead of 3 for b when a is 24. But it must be acknowledged that their figures possess at least as much of the rectilinear character as the corresponding one in the above diagram, which is not particularly successful; and in which, by being cut rather too deeply, is added to other probable errors the inequality produced by the change of position which

Fig. 40.

the edge of the tool makes in its revolution. Since, however, the formula was deduced mathematically by the author of the article "Trochoidal Curves,"—which has been already mentioned as the basis of the present paper,—and since it affords fairly satisfactory results in all cases,—it is only reasonable to assume its general accuracy, and to regard the ratio $\frac{b}{a} = \frac{1}{8}$ as only a convenient approximation for that of $\frac{b}{a} = \frac{1}{n^2}$, which in the

case of external four-looped figures becomes $\frac{1}{9}$ instead of $\frac{1}{8}$. To compare the two methods, the following experiment, fig. 40, was tried on as large a scale as the instrument permits.

$$a = 99, b = 11 \quad (\frac{b}{a} = \frac{1}{n^2} = \frac{1}{9})$$

$$a = 99, b = 12{\cdot}4 \ (\frac{b}{a} = \frac{1}{8})$$

The two curves intersect twice at each corner of the "square," and the line which is exterior to the other at the side, becomes therefore the interior of the two at the corner. This line is the one described with $b = 12{\cdot}4$, and is perhaps more nearly straight than the other between the points of intersection. On the other hand, the line described with $b = 11$ is a little hollow at the sides, but comes more into the corners, and may be considered to be approximately straight for a greater distance than the former. For all practical purposes of ornamentation, of course, either ratio may be employed. After all, it is not an easy matter to know the *exact* excentricities in use, nor to obtain a central adjustment in the first instance; though the dot, which may be here distinguished in the middle of the figure, proves that to have been fairly correct on this occasion.

7. ($x = 60, y = 36$, two carriers), V = 5, loops internal, $n = 6$. Fig. 41.

$a = 38{\cdot}9, b = \frac{a}{n^2} = 1{\cdot}1$, rectilinear

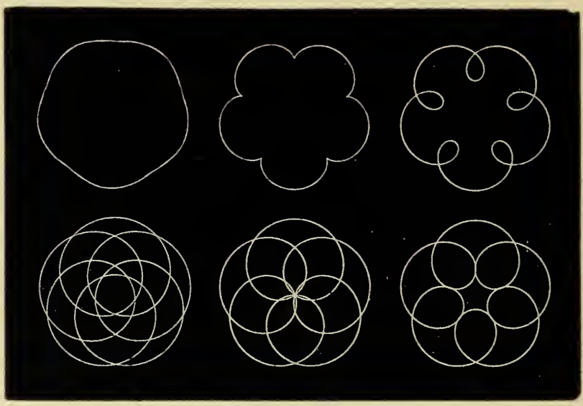
$a = 34{\cdot}3, b = \frac{a}{n} = 5{\cdot}7$, cusps

$a = 29, \quad b = 11$, loops

$a = 23{\cdot}5, \quad b = 16{\cdot}5$, „ touch

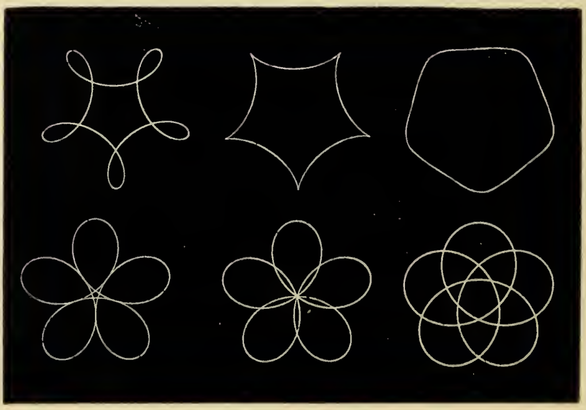
$a = 20$, $b = 20$, loops pass through centre
$a = 15$, $b = 25$, ,, intersect

Fig. 41.



8. ($x = 60$, $y = 36$, one carrier), $V = 5$, loops ex-ternal, $n = -4$. Fig. 42.

Fig. 42.



$a = 37{\cdot}7$, $b = \dfrac{a}{n^2} = 2{\cdot}3$, rectilinear

$a = 32$,	$b = \frac{a}{n} = 8$,	cusps	
$a = 27$,	$b = 13$,	loops	
$a = 20{\cdot}8$,	$b = 19{\cdot}2$,	,,	touch
$a = 20$,	$b = 20$,	,,	pass through centre
$a = 15$,	$b = 25$,	,,	intersect

It is interesting to remark that the same adjustments are required for the present case, where $V = 5$, $n = -4$, as were found to succeed in fig. 36, where $V = 3$, $n = 4$; and the same coincidence may have been observed between the adjustments for six loops external ($V = 6$, $n = -5$) and for four loops internal ($V = 4$, $n = 5$); also between those for four loops external ($n = -3$) and for two loops internal ($n = 3$). The phases of the curve clearly depend upon the value of n, and are independent of its positive or negative character, which latter is an indication solely of the directions in which the Flange and Frame are respectively moving.

For the sake of simplicity, therefore, the minus sign (which should properly be prefixed to the numerical values of n when the loops are external) will be omitted in future, unless there be some special reason for its retention.

CHAPTER III.

DEVELOPMENT OF "CIRCULATING" CURVES.

When such change wheels are employed as give to V, from the equation $V = \frac{3x}{y}$, a fractional value; n is also fractional, and the curve is no longer completed by one rotation of the pulley. The loops are now of the kind called "circulating," and pass through the same general forms as those already noticed. They intersect or recede from the centre, increase or diminish in magnitude, become cuspidated or polygonal, in dependence upon the same relative values of n, a, and b as those which determine the changes of the curve when of the more simple varieties appearing in the previous examples. In the following remarks (a) and (b) continue to represent the excentricities of Flange and of Frame, (x) and (y)* the numbers of teeth in the two change wheels, and (n) has the same signification and effect as before. The value of V when integral can never exceed 6, and when fractional can never reach it, since no change wheels larger than 60 or less than 30 are available in the present compact form of the instrument. And the more nearly any particular fractional value approaches to one of the whole numbers 2, 3, 4, 5, or 6, the more nearly will the course of the curve follow in its reduplication that which is produced when V is equal to the integer in question.

* See p. 13.

For example, when $x = 60$, $y = 32$, we have

$$V = \frac{3x}{y} = \frac{3 \times 60}{32} = \frac{45}{8} = 5{\cdot}62$$

$$\text{and } n\text{, when positive, } = V + 1 = \frac{53}{8}$$

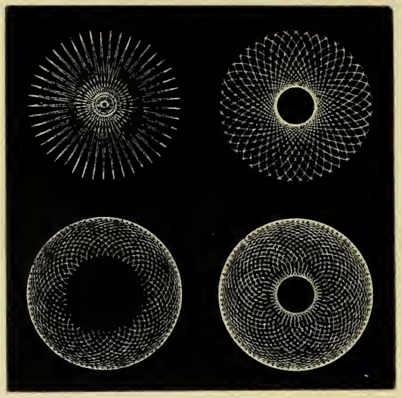
$$\text{and } n\text{, when negative, } = V - 1 = \frac{37}{8}$$

In accordance with the "planetary" explanation transcribed and discussed at a former page from the authority there quoted, n may take the form of the fraction $\frac{p}{q}$; and, when so expressed, there will be $(p \sim q)$ apocentres and pericentres (i.e. loops) if the motion is direct, and n positive, and $(p + q)$ loops when the motion is inverse and n negative. It has also been pointed out that q stands for the number of rotations of the Flange requisite to complete the curve. The present example confirms these statements: for when n is positive, there are $53 - 8$, or 45 loops; and, when negative, there are $37 + 8$, or still 45 loops. But it also shows that it is unnecessary to take n into account to find the number of loops produced or rotations required; since if $\frac{L}{R}$ denote the fractional value of V, the numerator (L) gives the number of loops, whether external or internal, and the denominator (R) the number of rotations of the pulley required to complete the curve.

With the change wheels named above, 45 loops are produced, and the value of V, being 5·62, shows that the curve partakes by repetition somewhat of the character of that with 6 simple loops. Again, when $x = 30$, and $y = 46$, there are also 45 loops; but the

value of V being now very nearly equal to 2, shows that the figure, according as it may be internal or external, will be composed of successive lines resembling a curve with two loops inwards,—or an ellipse. In the

Fig. 43.



former case R = 8, and the consecutive loops are formed within 8 of each other; in the latter, R = 23, and the consecutive loops are formed almost at opposite sides of the figure. The next two diagrams (figs. 43 and 44) explain this more clearly.

(x = 30, y = 46, one carrier) V = 1·95, loops (45) external, n = 0·95

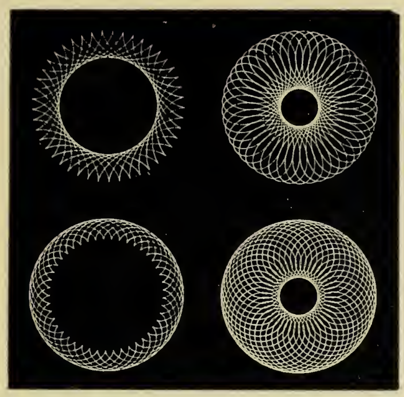
$$\left.\begin{array}{l} a = 19{\cdot}5,\ b = \dfrac{a}{n} = 20{\cdot}5 \\ a = 25,\quad b = 15 \end{array}\right\}\text{two upper figures (fig. 43)}$$

(x = 30, y = 46, two carriers) V = 1·95 loops (45) internal, n = 2·95

$$\left.\begin{array}{l} a = 32{\cdot}9,\ b = \dfrac{a}{n} = \quad 7{\cdot}1 \\ a = 25,\quad b = 15 \end{array}\right\}\text{two lower figures (fig. 43)}$$

Fig. 44.—($x = 60$, $y = 32$, one carrier) V = 5·62, loops (45) external, $n = 4·62$

Fig. 44.



$$\left.\begin{array}{l} a = 32·9,\ b = \dfrac{a}{n} = 7·1 \\ a = 25,\quad b = 15 \end{array}\right\}\text{two upper figures (fig. 44)}$$

($x = 60$, $y = 32$, two carriers) V = 5·62, loops (45) internal, $n = 6·62$

$$\left.\begin{array}{l} a = 34·8,\ b = \dfrac{a}{n} = 5·2 \\ a = 25,\quad b = 15 \end{array}\right\}\text{two lower figures (fig. 44)}$$

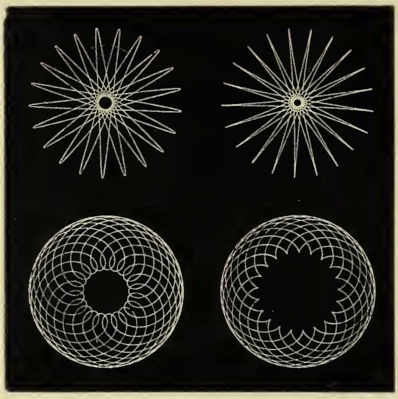
Another example of the differing effects produced where the number of loops is the same, but the value of V is affected by the substitution of other change wheels, may be had by comparing figs. 45 and 46, the loops being 21 in both instances.

($x = 42$, $y = 60$, one carrier) V = 2·1, loops (21) external, $n = 1·1$

$$\left.\begin{array}{l} a = 21·9,\ b = \dfrac{a}{n^2} = 18·1 \\ a = 21,\quad b = \dfrac{a}{n} = 19 \end{array}\right\}\text{two upper figures (fig. 45)}$$

The path of the tool has been almost entirely a straight line in the former of these, as has been shown to be more or less the case when the proportions of the

Fig. 45.



two excentricities are such that $\frac{a}{b} = n^2$. And the more closely V approaches to 2 (n nearly $= -1$), the more nearly rectilinear the curve will be under these conditions of adjustment. A similar approximation to the internal two-looped figure in its entirety will be remarked while the tool is describing a curve, with reversed motion (n nearly $= 3$), and with the same value for V.

($x = 42$, $y = 60$, two carriers) $V = 2{\cdot}1$, loops (21) internal, $n = 3{\cdot}1$

$$\left.\begin{array}{l} a = 26, \qquad b = 14 \\ a = 30{\cdot}25,\ b = \dfrac{a}{n} = 9{\cdot}75 \end{array}\right\}\begin{array}{c}\text{two lower figures} \\ \text{(fig. 45)}\end{array}$$

But here the effect is lost by the time the curve is completed. The general aspect of all specimens of the same class of *internal* loops—whether the loops be in contact, separated, or intersecting—is pretty much

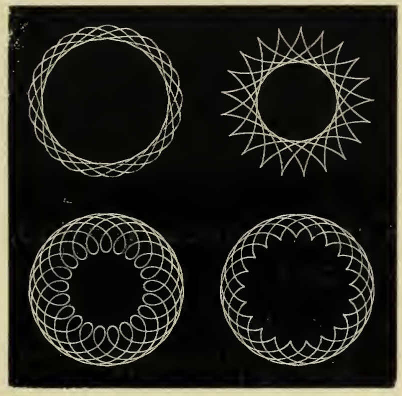
alike, provided the numbers of loops be not widely different. And the distinctive features attainable by altering the change wheels are generally much more striking when the loops are external; that is, when one "carrier" only is employed.

Fig. 46.—(x = 42, y = 30, one carrier) V = 4·2, loops (21) external, n = 3·2

$$\left.\begin{array}{l} a = 36{\cdot}5,\ b = \dfrac{a}{n^2} = 3{\cdot}5 \\ a = 30{\cdot}5,\ b = \dfrac{a}{n} = 9{\cdot}5 \end{array}\right\}\text{two upper figures}$$

The former is composed of a succession of lines, in continuation, each of which is of the character of the "square" (figs. 39 and 40) with a slightly wider angle; and if V had been more nearly equal to 4 than it is, the resemblance would, of course, have been greater.

Fig. 46.

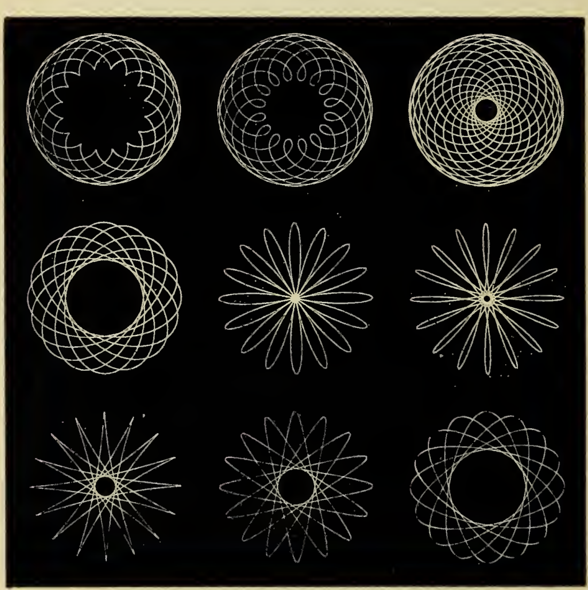


(x = 42, y = 30, two carriers), V = 4·2, loops (21) internal, n = 5·2

$$\left.\begin{array}{l} a = 30,\quad b = 10 \\ a = 33{\cdot}7,\ b = \dfrac{a}{n} = 6{\cdot}3 \end{array}\right\}\text{two lower figures (fig. 46)}$$

The next diagram (fig. 47) is a moderately good illustration of the resources of the Epicycloidal Cutting Frame, showing how considerably the design is affected by varying the excentricities (*a*) and (*b*) and the direction of motion without any change being made in the wheels employed.

Fig. 47.



(x = 32, y = 42, two carriers) V = 2·28, loops (16) internal, n = 3·28

$$\left.\begin{array}{ll} a = 30{\cdot}7, & b = \dfrac{a}{n} = 9{\cdot}3 \\ a = 28, & b = 12 \\ a = 17, & b = 23 \end{array}\right\}\text{three upper figures (fig. 47).}$$

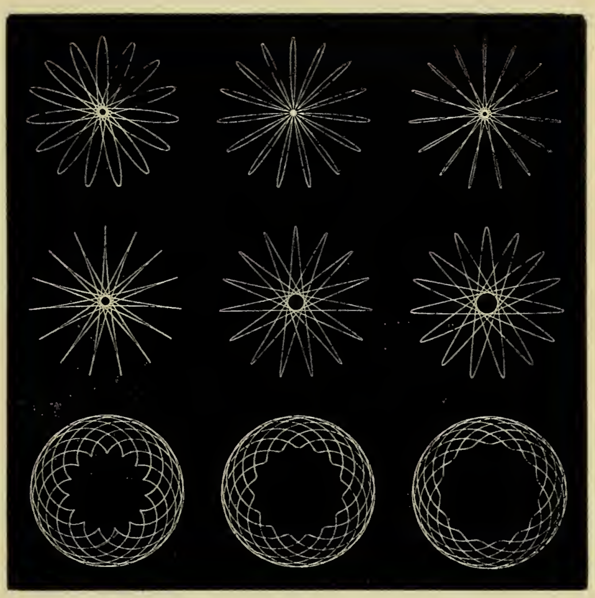
(x = 32, y = 42, one carrier) V = 2·28, loops (16) external, n = 1·28 (fig. 47)

$$\left.\begin{array}{l} a = 10,\ b = 30 \\ a = 20,\ b = 20 \\ a = 21,\ b = 19 \end{array}\right\}\text{three middle figures (fig. 47)}$$

$$\left.\begin{array}{ll} a = 22{\cdot}7, & b = \dfrac{a}{n} = 17{\cdot}3 \\ a = 24{\cdot}8, & b = \dfrac{a}{n^2} = 15{\cdot}2 \\ a = 30, & b = 10 \end{array}\right\}\text{three lower figures (fig. 47)}$$

The next figure is intended to show how small an alteration in the proportions of (a) and (b) will often make a considerable difference in the resulting curve.

Fig. 48.



($x = 30$, $y = 48$, one carrier) $V = 1{\cdot}87$, loops (15) external, $n = 0{\cdot}87$ (fig. 48)

$$\left.\begin{array}{ll} a = 21, & b = 19 \\ a = 20, & b = 20 \\ a = 19{\cdot}5, & b = 20{\cdot}5 \end{array}\right\}\text{three upper figures (fig. 48)}$$

$$\left.\begin{array}{ll} a = 18{\cdot}9, & b = \dfrac{a}{n} = 21{\cdot}1 \\ a = 18, & b = 22 \\ a = 17, & b = 23 \end{array}\right\}\text{three middle figures (fig. 48)}$$

The change wheels now in use give the lowest value for V which the instrument at present admits, viz. 1·87; and it will be at once perceived, on making the calculation, that the smaller n (and therefore V) may be, the wider will be the space occupied by the curve when reduced to the cuspidated form, and the bolder will be the cusps themselves.

The finely pointed star which stands first in the middle line (fig. 48) exemplifies this for external loops, and the figure below it shows the same effect internally.

When the cusps are well marked, as here (loops internal), the approximate figures, such as the two which follow on the lowest line, assume a wavy outline.

(x = 30, y = 48, two carriers) V = 1·87, loops (15) internal, n = 2·87 (fig. 48)

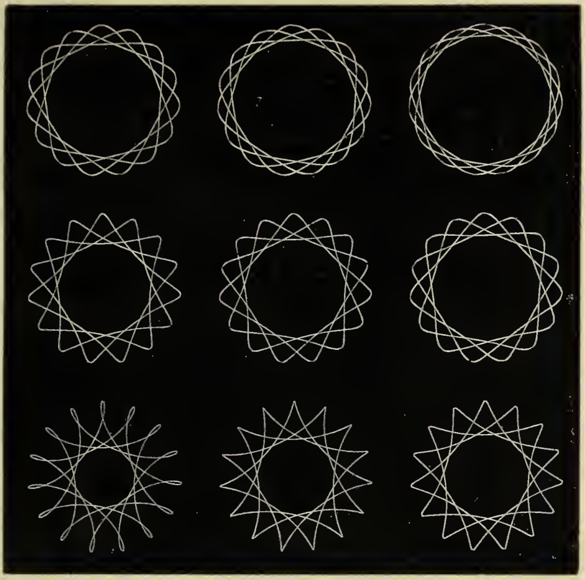
$$\left.\begin{array}{l} a = 29{\cdot}8,\ b = \dfrac{a}{n} = 10{\cdot}2 \\ a = 31{\cdot}5,\ b = 8{\cdot}5 \\ a = 33,\quad b = 7 \end{array}\right\}\text{three lower figures (fig. 48)}$$

The same number of loops, 15, with a much higher value for V, gives, at the earlier development of external loops, curves of the following character:

Fig. 49. (x = 40, y = 32, one carrier) V = 3·75, loops (15) external, n = 2·75.

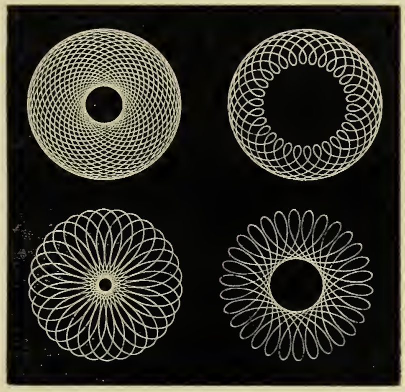
The figure occupying the narrowest space was described with a = 37, b = 3; and subsequently b was increased by *one* division (·01 inch) and a was diminished by the same amount.

Fig. 49.



A few specimens of loops of higher numbers may be acceptable.

Fig. 50.



$(x = 60, y = 42)$ V = 4·28, loops (30) fig. 50.

$\left.\begin{array}{l} a = 15,\ b = 25 \\ a = 32,\ b = 8 \end{array}\right\}$ two upper figures (internal)

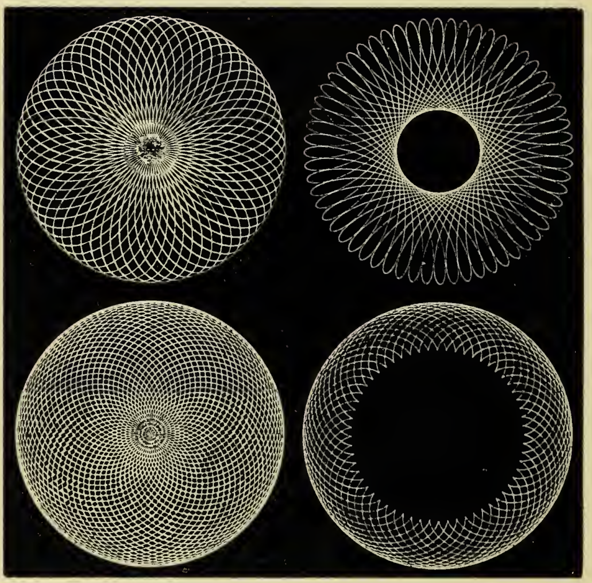
$\left.\begin{array}{l} a = 23,\ b = 17 \\ a = 27,\ b = 13 \end{array}\right\}$ two lower figures (external) fig. 50

$(x = 40, y = 34)$ $V = 3{\cdot}53$, loops (60). Fig. 51.

$\left.\begin{array}{l} a = 35,\ b = 35 \\ a = 46,\ b = 24 \end{array}\right\}$ two upper figures (external) fig. 51

$\left.\begin{array}{l} a = 35,\ b = 35 \\ a = 57,\ b = 13 \end{array}\right\}$ two lower figures (internal) ,,

Fig. 51.



These more numerous loops, with equal excentricities of Flange and Frame, produce, in a few seconds, patterns in "engine turning" such as the above; the figure being more fully covered when the loops turn inwards. It is not, however, to be recommended that the two excentricities should be quite equal; for, as in the present engravings, so many intersections at the centre make that part of the figure indistinct.

Further examples of curves, taken singly, will be

unnecessary. The amateur will find pleasure in verifying for himself the results which, from the explanations already offered, he will readily anticipate as belonging to any given pair of change wheels, and to any assigned adjustments of the instrument. But to facilitate the selection of such wheels as will best produce a desired effect, the following Tables have been computed.

Table I. shows the value (V), expressed both fractionally and in decimals, of the whole train of wheels corresponding in each case to the several change wheels of the dimensions given. At those intersections of the columns where no figures appear, the combination of wheels, whose value would otherwise occupy the blank space, is impracticable from the construction of the instrument. The numerator of each fraction denotes the number of loops produced; the denominator expresses the number of rotations of the pulley requisite to complete the curve, and thus gives a rough comparative indication of the resulting figure.

In Table II., the numerator of the fraction appears in the column headed "Loops," arranged in the order of their numbers, and the denominator is found in the column marked R.

Table III. contains the same elements as the two preceding tables, the column of entry being now the values of V, placed in order of magnitude. In all three, (x) and (y) represent the two change wheels, (x) being that which is first placed upon the removable arbor.

Remembering that $n = V \pm 1$, these tables indicate at a glance what will be the general result of using any two of the change wheels provided; and, conversely, what change wheels should be selected in order to obtain loops of any required character and number.

TABLE I.

$y=$	$x=60$	$x=48$	$x=46$	$x=44$	$x=42$	$x=40$	$x=38$	$x=36$	$x=34$	$x=32$	$x=30$	$=y$
60	×	$\frac{12}{5}=2{\cdot}4$	$\frac{23}{10}=2{\cdot}3$	$\frac{11}{5}=2{\cdot}2$	$\frac{21}{10}=2{\cdot}1$	2	$\frac{19}{10}=1{\cdot}9$	×	×	×	×	60
48	$\frac{15}{4}=3{\cdot}75$	3	$\frac{23}{8}=2{\cdot}88$	$\frac{11}{4}=2{\cdot}75$	$\frac{21}{8}=2{\cdot}63$	$\frac{5}{2}=2{\cdot}5$	$\frac{19}{8}=2{\cdot}37$	$\frac{9}{4}=2{\cdot}25$	$\frac{17}{8}=2{\cdot}12$	2	$\frac{15}{8}=1{\cdot}87$	48
46	$\frac{90}{23}=3{\cdot}91$	$\frac{72}{23}=3{\cdot}26$	×	$\frac{66}{23}=2{\cdot}87$	$\frac{63}{23}=2{\cdot}74$	$\frac{60}{23}=2{\cdot}61$	$\frac{57}{23}=2{\cdot}48$	$\frac{54}{23}=2{\cdot}35$	$\frac{51}{23}=2{\cdot}22$	$\frac{48}{23}=2{\cdot}09$	$\frac{45}{23}=1{\cdot}95$	46
44	$\frac{45}{11}=4{\cdot}09$	$\frac{36}{11}=3{\cdot}27$	$\frac{69}{22}=3{\cdot}13$	×	$\frac{63}{22}=2{\cdot}86$	$\frac{30}{11}=2{\cdot}72$	$\frac{57}{22}=2{\cdot}59$	$\frac{27}{11}=2{\cdot}45$	$\frac{51}{22}=2{\cdot}32$	$\frac{24}{11}=2{\cdot}17$	$\frac{45}{22}=2{\cdot}04$	44
42	$\frac{30}{7}=4.28$	$\frac{24}{7}=3{\cdot}43$	$\frac{23}{7}=3{\cdot}28$	$\frac{22}{7}=3{\cdot}14$	×	$\frac{20}{7}=2{\cdot}86$	$\frac{19}{7}=2{\cdot}71$	$\frac{18}{7}=2{\cdot}57$	$\frac{17}{7}=2{\cdot}43$	$\frac{16}{7}=2{\cdot}28$	$\frac{15}{7}=2{\cdot}14$	42
40	$\frac{9}{2}=4{\cdot}5$	$\frac{18}{5}=3{\cdot}6$	$\frac{69}{20}=3{\cdot}45$	$\frac{33}{10}=3{\cdot}3$	$\frac{63}{20}=3{\cdot}15$	×	$\frac{57}{20}=2{\cdot}85$	$\frac{27}{10}=2{\cdot}7$	$\frac{51}{20}=2{\cdot}55$	$\frac{12}{5}=2{\cdot}4$	$\frac{9}{4}=2{\cdot}25$	40
38	$\frac{90}{19}=4{\cdot}74$	$\frac{72}{19}=3{\cdot}79$	$\frac{69}{19}=3{\cdot}63$	$\frac{66}{19}=3{\cdot}47$	$\frac{63}{19}=3{\cdot}32$	$\frac{60}{19}=3{\cdot}16$	×	$\frac{54}{19}=2{\cdot}84$	$\frac{51}{19}=2{\cdot}68$	$\frac{48}{19}=2{\cdot}53$	$\frac{45}{19}=2{\cdot}37$	38
36	5	4	$\frac{23}{6}=3{\cdot}83$	$\frac{11}{3}=3{\cdot}66$	$\frac{7}{2}=3{\cdot}5$	$\frac{10}{3}=3{\cdot}33$	$\frac{19}{6}=3{\cdot}17$	×	$\frac{17}{6}=2{\cdot}83$	$\frac{8}{3}=2{\cdot}66$	$\frac{5}{2}=2{\cdot}5$	36
34	$\frac{90}{17}=5{\cdot}29$	$\frac{72}{17}=4{\cdot}23$	$\frac{69}{17}=4{\cdot}06$	$\frac{66}{17}=3{\cdot}88$	$\frac{63}{17}=3{\cdot}7$	$\frac{60}{17}=3{\cdot}53$	$\frac{57}{17}=3{\cdot}35$	$\frac{54}{17}=3{\cdot}17$	×	$\frac{48}{17}=2{\cdot}82$	$\frac{45}{17}=2{\cdot}65$	34
32	$\frac{45}{8}=5{\cdot}62$	$\frac{9}{2}=4{\cdot}5$	$\frac{69}{16}=4{\cdot}31$	$\frac{33}{8}=4{\cdot}12$	$\frac{63}{16}=3{\cdot}94$	$\frac{15}{4}=3{\cdot}75$	$\frac{57}{16}=3{\cdot}56$	$\frac{27}{8}=3{\cdot}37$	$\frac{51}{16}=3{\cdot}18$	×	$\frac{45}{16}=2{\cdot}81$	32
30	6	$\frac{24}{5}=4{\cdot}8$	$\frac{23}{5}=4{\cdot}6$	$\frac{22}{5}=4{\cdot}4$	$\frac{21}{5}=4{\cdot}2$	4	$\frac{19}{5}=3{\cdot}8$	$\frac{18}{5}=3{\cdot}6$	$\frac{17}{5}=3{\cdot}4$	$\frac{16}{5}=3{\cdot}2$	×	30

TABLE II.

Loops.	R	V	*x*	*y*	Loops.	R	V	*x*	*y*	Loops.	R	V	*x*	*y*
2	1	2	32	48	21	8	2·63	42	48	51	22	2·32	34	44
3	1	3	48	48	,,	10	2·1	42	60	,,	23	2·22	34	46
4	1	4	48	36	22	5	4·4	44	30	54	17	3·17	36	34
5	1	5	60	36	,,	7	3·14	44	42	,,	19	2·84	36	38
,,	2	2·5	40	48	23	5	4·6	46	30	,,	23	2·85	36	46
6	1	6	60	30	,,	6	3·83	46	36	57	16	3·56	38	32
7	2	3·5	42	36	,,	7	3·14	46	42	,,	17	3·35	38	34
8	3	2·6	32	36	,,	8	2·88	46	48	,,	20	2·85	38	40
9	2	4·5	60	40	,,	10	2·3	46	60	,,	22	2·59	38	44
,,	4	2·25	36	48	24	5	4·8	48	30	,,	23	2·48	38	46
10	3	3·3	40	36	,,	7	3·43	48	42	60	17	3·53	40	34
11	3	3·6	44	36	,,	11	2·17	32	44	,,	19	3·17	40	38
,,	4	2·75	44	48	27	8	3·37	36	32	,,	23	2·61	40	46
,,	5	2·2	44	60	,,	10	2·7	36	40	63	16	3·94	42	32
12	5	2·4	32	40	,,	11	2·45	36	44	,,	17	3·7	42	34
15	4	3·75	40	32	30	7	4·28	60	42	,,	19	3·32	42	38
,,	7	2·14	30	42	,,	11	2·72	40	44	,,	20	3·15	42	40
,,	8	1·87	30	48	33	8	4·12	44	32	,,	22	2·86	42	44
16	5	3·2	32	30	,,	10	3·3	44	40	,,	23	2·74	42	46
,,	7	2·28	32	42	36	11	3·27	48	44	66	17	3·88	44	34
17	5	3·4	34	30	45	8	5·62	60	32	,,	19	3·47	44	38
,,	6	2·83	34	36	,,	11	4·09	60	44	,,	23	2·87	44	46
,,	7	2·43	34	42	,,	16	2·81	30	32	69	16	4·31	46	32
,,	8	2·12	34	48	,,	17	2·65	30	34	,,	17	4·06	46	34
18	5	3·6	48	40	,,	19	2·37	30	38	,,	19	3·63	46	38
,,	7	2·57	36	42	,,	22	2·04	30	44	,,	20	3·45	46	40
19	5	3·8	38	30	,,	23	1·95	30	46	,,	22	3·13	46	44
,,	6	3·17	38	36	48	17	2·82	32	34	72	17	4·23	48	34
,,	7	2·71	38	42	,,	19	2·53	32	38	,,	19	3·79	48	38
,,	8	2·37	38	48	,,	23	2·09	32	46	,,	23	3·26	48	46
,,	10	1·9	38	60	51	16	3·18	34	32	90	17	5·29	60	34
20	7	2·86	40	42	,,	19	2·68	34	38	,,	19	4·74	60	38
21	5	4·2	42	30	,,	20	2·55	34	40	,,	23	3·91	60	46

TABLE III.

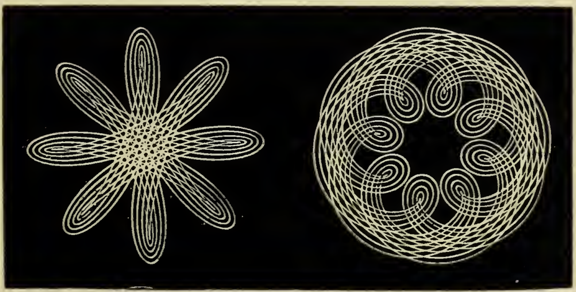
V	Loops.	x	y	V	Loops.	x	y	V	Loops.	x	y
1·87	15	30	48	2·7	27	36	40	3·45	69	46	40
1·9	19	38	60	2·71	19	38	42	3·47	66	44	38
1·95	45	30	46	2·72	30	40	44	3·5	7	42	36
2	2	32	48	2·74	63	42	46	3·53	60	40	34
2·04	45	30	44	2·75	11	44	48	3·56	57	38	32
2·09	48	32	46	2·81	45	30	32	3·6	18	48	40
2·1	21	42	60	2·82	48	32	34	3·63	69	46	38
2·12	17	34	48	2·83	17	34	36	3·66	11	44	36
2·14	15	30	42	2·84	54	36	38	3·7	63	42	34
2·17	24	32	44	2·85	57	38	40	3·75	15	60	48
2·2	11	44	60	2·86	20	40	42	3·79	72	48	38
2·22	51	34	46	2·86	63	42	44	3·8	19	38	30
2·25	9	36	48	2·87	66	44	46	3·83	23	46	36
2·28	16	32	42	2·88	23	46	48	3·88	66	44	34
2·3	23	46	60	3	3	48	48	3·91	90	60	46
2·32	51	34	44	3·13	69	46	44	3·94	63	42	32
2·35	54	36	46	3·14	22	44	42	4	4	48	36
2·37	19	38	48	3·15	63	42	40	4·06	69	46	34
2·37	45	30	38	3·16	60	40	38	4·09	45	60	44
2·4	12	48	60	3·17	19	38	36	4·12	33	44	32
2·43	17	34	42	3·17	54	36	34	4·2	21	42	30
2·45	27	36	44	3·18	51	34	32	4·23	72	48	34
2·48	57	38	46	3·2	16	32	30	4·28	30	60	42
2·5	5	40	48	3·26	72	48	46	4·31	69	46	32
2·53	48	32	38	3·27	36	48	44	4·4	22	44	30
2·55	51	34	40	3·28	23	46	42	4·5	9	60	40
2·57	18	36	42	3·3	33	44	40	4·6	23	46	30
2·59	57	38	44	3·32	63	42	38	4·74	90	60	38
2·61	60	40	46	3·33	10	40	36	4·8	24	48	30
2·63	21	42	48	3·35	57	38	34	5	5	60	36
2·65	45	30	34	3·37	27	36	32	5·29	90	60	34
2·66	8	32	36	3·4	17	34	30	5·62	45	60	32
2·68	51	34	38	3·43	24	48	42	6	6	60	30

CHAPTER IV.

INVESTIGATION OF FORMULÆ FOR CORRECTING THE OBLIQUITY DUE TO THE RADIAL ACTION OF THE FLANGE.

HITHERTO, in all the illustrations accompanying these remarks, each figure has been composed of a single line. But in ornamental design the effect is generally improved by either repeating the same curve with varying excentricities, or by associating with the first, other curves of different form and direction. It is requisite that these successive curves should assume symmetrical positions, the whole having one common axis, or in part differing therefrom according to some definite and prearranged system. But here we are met with the difficulty that although, while the value of (a) remains fixed, (b) may be altered to any extent without affecting the position of the new curve with reference to the axis of the first, yet an alteration in the value of (a), whether (b) be disturbed or not, at once changes the relation

Fig. 52.



which previously subsisted between that axis and the course of the next curve. For example, in fig. 52, figures of 8 loops, both external and internal, have been placed separately in parallel lines, by increasing the excentricity of the Frame 3 divisions at a time, while the excentricity of the Flange remained constant; and no change occurred in the angular positions. But in fig. 53, the transition is very marked from the vertical character of the cusps which form the boundary of the pattern, to the inclined position of the inmost looped

Fig. 53.

curve. The effect, as it stands, is not unpleasing, though it plainly exhibits the kind of discrepancy to which attention is now invited, and which in most cases is preferably corrected. The figure was described with six loops inwards; the value of (*b*) was 11 throughout, and (*a*) was diminished by 5 divisions at a time from 80 to 20. In the converse form, fig. 54, (*b*) was kept at 16, while (*a*) extended, by the same intervals as in the last figure, from 20 to 75. The angular deviation is here much less apparent, as is invariably the

case with external loops as compared with internal; and it happens, as will be seen subsequently, that the

Fig. 54.

deviation attains its greatest extent in the six-looped (internal) figure, and its least when the figure has six external loops. The following example, fig. 55, shows

Fig. 55.

the inequality more plainly. All the eleven loops pass through the common centre of each of the seven curves,

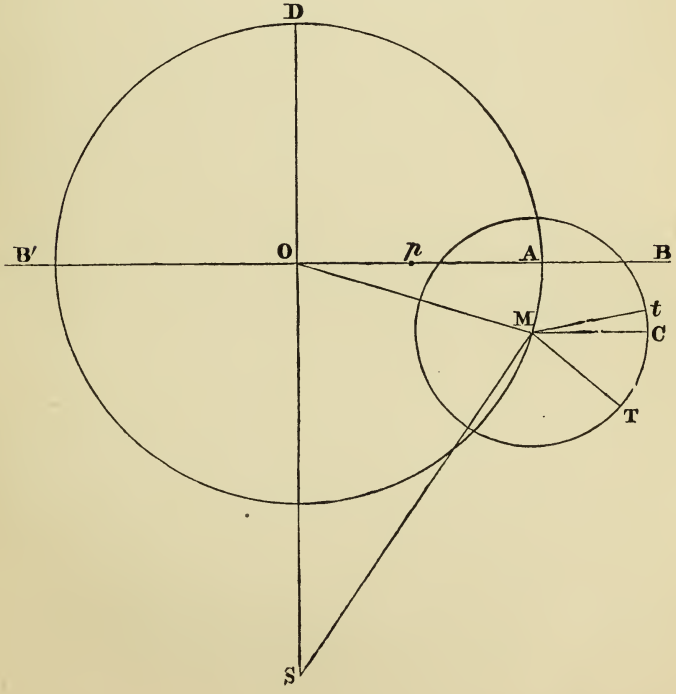
(*a*) being equal to (*b*) for each adjustment; but it will be observed that the inner loops all lie more closely towards one side than the other of the largest and exterior loop.

A means of counteracting this disturbance of position caused by the movement of the Flange upon its centre during the alteration of its excentricity, is provided by the tangent wheel and screw seen at the back of the instrument in the engraving which appears in the frontispiece. The micrometer head of this screw enables the central wheel of 64 teeth, (which is always stationary while the rest are revolving round it,) to be moved on its axis through the $\frac{1}{4800}$ part of its circumference or more; and the graduations of the micrometer are sufficiently wide to permit an easy estimation of subdivisions. The amount of disturbance, and therefore the amount of compensation required, is independent of the values of (*a*) and (*b*), depending solely upon the value of the train according to the change wheels employed. A table of approximate corrections—derived experimentally—for some loops of the lower numbers, has been usually furnished with the instrument by the makers, and for all practical purposes that table is sufficiently exact so far as it extends. But it is believed that the general expressions, deduced from the considerations which follow, will be found to be more strictly accurate, as well as more convenient in application, and to hold good for all values of (*x*) and (*y*) whatever.

Resuming the theoretical view of the subject, and again looking upon the excentricity of the Flange as the radius of "the deferent," and the excentricity of the Frame as the radius of "the epicycle," we see that the radial action of the Flange changes the position of the

point M upon the deferent, and *also* causes a certain amount of rotation in the wheels which form the latter part of the train. The combined effect is, that the "initial position" (of the Flange as parallel to the lathe bearers, and the Frame perpendicular to the Flange) is disturbed; and that the apocentres and pericentres of the curve no longer occur on the same radii as before of the apocentral and pericentral circles. The same kind of displacement arises, whether the curve be an ellipse, or one of the various figures with internal or external loops or cusps, resulting from other concurrent values of n, a, and b; and the compensating adjustment is applied in the same manner by the micrometer screw of the tangent wheel, though differing in degree in all the several cases.

Fig. 56.



In fig. 56 above, let B′ B, passing through O, be the *datum line*, or axis, with respect to which the curve is to be symmetrically placed. It will, therefore, bisect an apocentre at each end, or an apocentre at one end and a pericentre at the other, according as the number of loops contained in the curve is even or uneven. Let O be the centre of the instrument, and therefore of the deferent circle M A D; and let S be the centre of the stud on which the Flange radiates. The figure represents the Flange vertical in the line D O S, and the Frame is, therefore, supposed to be horizontal, in the line B′ B.

Take O *p* for the radius of the epicycle; that is for the amount of excentricity to be given to the Frame. Then, while the Flange is central, the circle which is to become the epicycle is central also.

Now take O A for the radius of the deferent, that is for the excentricity of the Flange. Then, S M being equal to S O, the movement of the Flange will depress that radius into the position O M: and the radius of the epicycle, instead of being brought to A B, will coincide with some radius of the circle T C *t*. Two cases here present themselves.

I. If the value of the latter part of the train, which includes the change wheels, i. e. from S on the axis of the Flange to M on the axis of the Frame, be such that there is no rotation whatever of this last axis, while the excentricity of the Flange is being altered; then the position of P in the circumference of the epicycle is not disturbed; and P is brought by the action of the Flange to the point C, M C being, as in previous figures, parallel to O A B.

II. But if the value of the short train from S to M be not equal to that just supposed, and some amount of

rotation of the last axis *does* take place during the alteration of the excentricity of the Flange, P will no longer coincide with C, after the Flange has been moved, but will be found at some other point of the circumference of the epicycle T C *t*.

Referring to the instrument for an elucidation of these two cases, it will be seen by experiment that,

(i.) When V = 2, the radial movement of the Flange alone produces no change whatever in the inclination of the Frame: the effect, under these conditions, is merely to change the position of the axis of the Frame with reference to the centre of the instrument. In fact, if the actuating screw be withdrawn, and the pulley be kept stationary, the Flange may be moved by hand backwards, upon its stud as centre, for nearly three quarters of a circle, until prevented from further advance by the projection of other parts of the mechanism: and, during the whole time, the Frame remains parallel to the position which it first occupied. [Case I.]

But, (ii.), when V has any other value than 2, the radial movement of the Flange, during its adjustment, *does* impart some degree of rotation to the axis of the Frame, and induces a corresponding change in its inclination, besides changing the position of the axis itself by a "motion of translation," with reference to the general centre. [Case II.]

It is evident that the fact of the epicycle being direct or retrograde, that is whether two carriers are in operation or only one, will make no difference in the amount of *displacement* (though it affects materially the corresponding amount of *compensation*), for the wheels on S, and those behind it, remain stationary while the Flange is moved; and the Flange is only moved for addition of excentricity in one direction.

Now a pericentre is formed when the two excentricities of Flange and Frame are in one straight line $(a - b)$, and are opposed to one another; and an apocentre is formed when they combine, also in one straight line $(a + b)$. To preserve the symmetry of the curve with respect to others which may be included in the design, a combination or opposition of excentricities must take place in B′ B, which has been assumed as the axis or datum line for all the intended curves.

Therefore, what is required in order to correct the disturbance which has been shown to exist, is that such an amount of counter-revolution may be given to P, after fixing the excentricity of the Flange, and before tracing the curve, as will leave P at such a distance from C (say at T or *t*) that when, by the rotation of the pulley, M has moved up to A, P shall also have arrived at B, moving either positively or negatively (i. e. upwards from T, or downwards from *t*) according as the epicycle is direct or retrograde: so that M and P may cross B′ B simultaneously. For then P will be in apocentre at the moment of passing the datum line, and will form the extremity of a loop at that instant, and in that line.

But P moves n times as fast as M; therefore, if the epicycle be direct, P must be moved in correction backwards to T, until the angle C M T $= n \cdot$ M O A: and if the epicycle be retrograde, the movement of P in correction must be forwards to *t*, the angle C M *t* being again $= n \cdot$ M O A. [It will be borne in mind that n has *not the same value* in these two instances.] And, if we can ascertain *whereabouts, on the circumference of the epicycle,* P *has been left* after the Flange has been moved when receiving its excentricity, and can also find the value of the angle M O A, then, since n is known (being $V + 1$, or $V - 1$), the angle C M T, or

C M *t*, will be known also: and it only remains to give to the axis of the Eccentric Frame, by the micrometer screw of the tangent wheel, such a fraction of a revolution as is equal to that angle. The act of turning this micrometer screw, to which for that purpose a light winch handle is fitted, gives rotation to all the wheels of the train, without reference to the pulley: and therefore, whatever part of a turn is given to the tangent wheel is transmitted V times to the axis of the Frame; which, under these circumstances, describes, by the point of the tool, an arc of a circle coincident with the epicycle: That is, *the rotation of the tangent wheel alters* the inclination of the Eccentric Frame, and changes the *position of* P *upon the epicycle*, causing it to move through the required angle C M T or C M *t*.

The angle through which the Frame has thus to be moved, as a process of compensation, is *the angle of correction*, whose magnitude we may now endeavour to ascertain. The former of the two cases stated above, viz., where the action of the Flange produces no change in the place of P on the epicycle, is the simpler of the two, and the correction for this smaller amount of disturbance will be the more readily obtained.

I. It is shown by Professor Willis in the work already cited,* that when an epicyclic train of three wheels, of which the first and third are equal, and the first is fixed, is carried round by an arm attached, as on a pivot, to the centre of the first wheel, the third wheel does not rotate, but is carried round in one position, so that any radius always remains parallel to itself. Now the wheel (60), on the axis of S, is the first of what has been called "the latter part of the train;" and it remains fixed while the Flange is being moved

* Principles of Mechanism.

upon its stud S. Also, when the wheels $x = 32$, $y = 48$, are in use, which give the two-looped figure, or the ellipse, we have $\frac{60}{48} \times \frac{32}{40} = 1$ for the value of the train from S to M. Therefore, in this case (which is when $V = 2$ and $n = 3$ or 1), we have an instance of the kind of epicyclic train in question, viz., where there is no gain or loss of velocity between the wheels on the first axis and on the third; and P remains constantly at C, during the adjustment of the Flange, whatever be the length of O M.

Referring to fig. 56, P is at C, and it is desired to bring it to such a point (T, if the epicycle be direct, and a two-looped curve is to be produced: *t*, if the epicycle be retrograde, and an ellipse is to be described), that C M T, or C M *t*, which may be called the "angle of correction," shall be equal to $n \cdot$ M O A.

Now M O A $= \frac{1}{2}$ O S M, whatever be the value of O M.

For, the exterior angle D O M is equal to the two interior angles O M S and O S M.

$$\text{or,} \quad \mathrm{DOA} + \mathrm{MOA} = \mathrm{MOS} + \mathrm{OSM} \ (\text{since } \mathrm{OS} = \mathrm{OM}).$$

But D O A is a right angle, and the two angles M O S, M O A, are together equal to a right angle.

$$\therefore (\mathrm{MOS} + \mathrm{MOA}) + \mathrm{MOA} = \mathrm{MOS} + \mathrm{OSM}$$

$$\therefore 2\,\mathrm{MOA} = \mathrm{OSM}; \text{ or } \mathrm{MOA} = \tfrac{1}{2}\,\mathrm{OSM}.$$

Also, C M T, which $= n \cdot$ M O A, is therefore $= n \cdot \dfrac{\mathrm{OSM}}{2}$.

To find the value of O S M, we have O M, the excentricity of the Flange, indicated, as the chord of the arc, in hundredths of an inch; and O S, which is known, by actual measurement, to be equal to 2 inches.

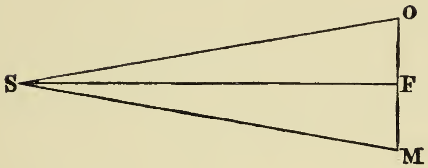
$\dfrac{\mathrm{OM}}{\mathrm{OS}}$ is the chord of the angle O S M to the radius O S:

and in the right-angled triangle O S F (fig. 57), O F being half O M,

$$\text{we have } \frac{\text{O F}}{\text{O S}} = \sin \text{O S F}$$

$$= \sin \frac{\text{O S M}}{2}$$

Fig. 57.



$$\therefore \log \sin \frac{\text{O S M}}{2} = 10 + \log \text{O F} - \log \text{O S}.$$

Suppose O M to be equal to one division of the Flange, = 1; then O S = 200; both quantities being expressed in hundredths of an inch.

Then O F = 0·5, whose logarithm,* + 10 = 9·69897
and O S = 200, " " = 2·30103

$$\therefore \log \sin \frac{\text{O S M}}{2} = 7{\cdot}39794$$

$$\text{and } \frac{\text{O S M}}{2} = 0^\circ 8' 36''$$

In determining an angle to so small a radius as 2 inches, seconds of arc will be practically inappreciable: we may therefore say, with sufficient accuracy, that

$$\frac{\text{O S M}}{2} = 0^\circ\ 9'.$$

* If the reader should not be fully acquainted with the use of Logarithms, he will find all information upon the subject, and a very handy set of those tables, in Law's *Rudimentary Treatise on Logarithms, and Mathematical Tables*, one of the cheap series published by Weale. (2*s.* 6*d.* Virtue & Co.)

Let the "angle of correction," be denoted by θ. Now $\theta = n \cdot$ MOA (page 60); and the value of n is 3 for the internal two looped figure; and 1 for the ellipse.

$$\text{Therefore, in the former case, } \theta = 3 \times 9' = 27';$$
$$\text{and, in the latter, } \theta = 1 \times 9' = 9'.$$

These are the angles through which, when V = 2, the Eccentric Frame is to be moved on its axis, in order to correct the error of inclination caused by a radial adjustment of the Flange of one division, at any part of its graduations.

If the tangent wheel were to transmit this motion of correction without change, the above values of 9′ and 27′ would represent the arcs through which the tangent wheel would have to pass, in the two cases respectively. But, as already remarked, the rotation of the Frame, when thus effected through the whole train of wheels, is V times greater than the corresponding rotation of the tangent wheel which imparts the motion.

In the present instance V = 2: therefore the angles through which the tangent wheel must be moved, in order to produce the required corrections in the two different cases, are half the quantities just stated. That is to say, the "angle of correction" through which the tangent wheel must pass under the conditions supposed is 0° 13′·5 in the one case, and 0° 4′·5 in the other. Both these angles are to be applied, when excentricity is added to the Flange, by turning the Tangent Screw in the direction in which the reading of its graduations increases, and *vice versâ* when that excentricity is diminished. [See page 71.]

Now the circumference of the tangent wheel is divided micrometrically into (96 × 50 =) 4800 parts;

and 360°, divided by 4800, gives, for the angular value of each of those parts, 0° 4′·5. [See Note, page 74.]

One division therefore of the micrometer will compensate for the obliquity caused by moving the *Flange one division* when the instrument is arranged to produce ellipses or straight lines. And, since 13′·5 = 4′·5 × 3, it follows that, when with the same change wheels, and another "carrier," the curve has taken the form of a two-looped figure,—*three divisions* on the micrometer will compensate for *one on the Flange.*

II. We may next proceed to investigate the angle of correction in the second of the cases stated on pages 58 and 59: viz., when the value of the latter part of the train being no longer equal to 1, the axis of the Frame receives, in addition to its motion of translation, a certain amount of rotation, in consequence of the radiation of the Flange; and when P therefore forms an angle PMC with MC, instead of coinciding with it. (fig. 58.) There are two divisions of the present case:—

(i) where the value of the short train from S to M is *greater than unity*; and,

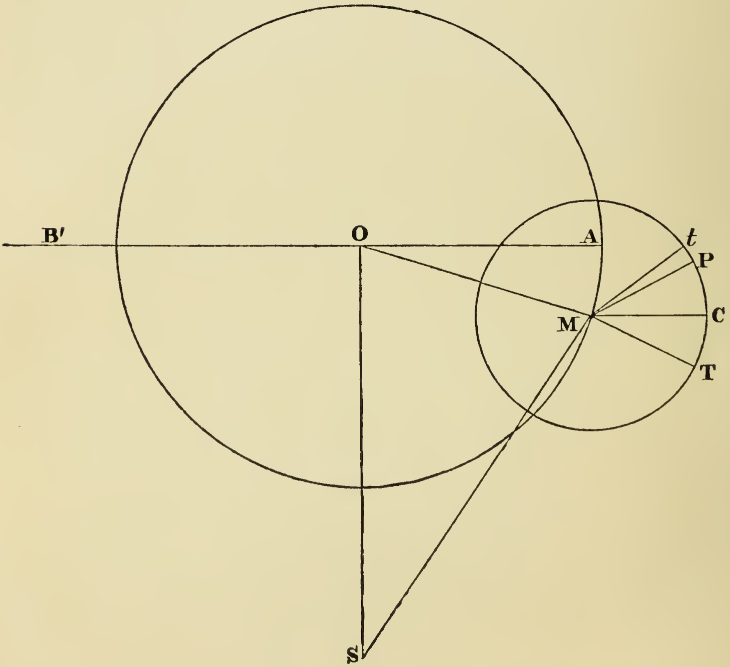
(ii) where that value is *less than unity.*

The former is by far the more frequent condition; but, although in the instrument in the writer's possession there are only three pairs of values for x and y which, as stated in the Tables, give a less value for V than 2; a few slight alterations, chiefly in countersinking heads of screws, would increase their number, possibly with increased range of ornamental effect. The three-looped figure given by $x = 30, y = 60$, for instance, possesses a very distinctive character.

(i) When the Flange is moved radially upon the stud S, during its excentric adjustment, the only wheels influenced by that movement are those previously

referred to as "the latter part of the train" from S to M. The value of this short train is denoted by $\frac{60}{y} \times \frac{x}{40}$, which is $= \frac{3x}{2y}$: therefore * for every turn of the Flange in adjustment (the pulley being meanwhile stationary) the axis of the Eccentric Frame makes $\left(1 - \frac{3x}{2y}\right)$ rotations; and, for such fraction of a turn as the Flange may make, the Frame axis will make $\left(1 - \frac{3x}{2y}\right)$ of the same fraction of a rotation.

Fig. 58.



Now the angle O S M (fig. 58) represents, in the manner in which it would be apparent to a person standing

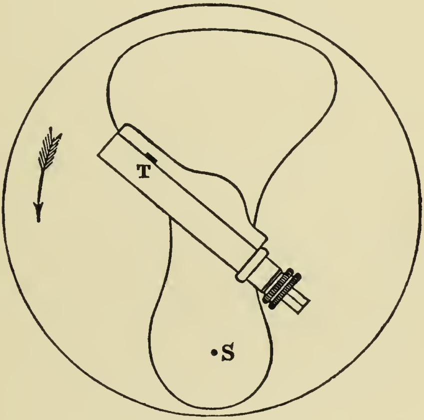
* Willis, *Principles of Mechanism*, edition 1870, p. 322.

behind the instrument, the fraction of a turn which the Flange is supposed to have made in receiving its excentricity: and the angle PMC is the corresponding fraction of a rotation made, during the same time, by the Frame axis, in consequence of the intervening wheels which constitute the "latter part of the train." And if these two angles were of the same kind, whether positive or negative, we should have

$$\text{PMC} = \left(1 - \frac{3x}{2y}\right) \text{OSM}.$$

But they are not of the same kind, as will be readily seen by analyzing, experimentally, the respective movements of Flange and Frame.

Fig. 59.



In fig. 59, supposed to be a partial front view of the instrument, let the centre of the Flange coincide with the common centre of the instrument; and let T be the point of the tool carried by the Eccentric Frame. It will be at once perceived, from the mechanical details of construction, that for the cutting edge of the tool

to be properly presented to the surface under ornamentation, the Frame must always revolve towards the left, i.e. in the direction of the arrow. Consequently, when "the epicycle is direct," and the loops are internal, and the Flange and Frame revolve in one direction, the pulley, carrying the Flange with it, must also turn in the direction of the arrow; and by the original hypothesis, this is the direction which it was agreed should be considered positive.

Now, excentricity is added to the Flange by moving it from right to left (as looked at from the front), that is, still in the direction of the arrow. But when the pulley is stationary, and the Flange receives a limited movement of this kind, the axis of the Frame receives a motion of translation in the same positive direction together with a motion of rotation (when such occurs, as it will, except when $V = 2$) in the opposite or negative direction.

That this is the fact is evident from the consideration of the general question of Epicyclic trains; * and, if the wheels $x = 60$, $y = 30$, which give the greatest attainable value (3) to the short train from S to M, be in use when the Flange receives a change of excentricity, the opposition of the two movements of Flange and Frame will be rendered practically very visible.

Therefore, when V is greater than 2 (whatever x and y may be, fulfilling that condition) we see that P is brought by the movement of the Flange in adjustment to some point *above* M C as drawn in fig. 58; and, consequently, if one of the angles P M C, O S M be considered positive, it is clear that the other must be considered negative. It follows that if we express one in terms of

* *Principles of Mechanism*, Willis; or *Elements of Mechanism*, Goodeve. *Text Books of Science*, Longmans, 1870.

the other, the sign of the coefficient must be changed, and we have

$$\text{PMC} = -\left(1 - \frac{3x}{2y}\right) \text{OSM}$$
$$= \left(\frac{3x}{2y} - 1\right) \text{OSM}$$

which gives us the information sought for as to the position on the epicycle in which P has been left by an assigned movement of the Flange in excentricity.

The same object has to be attained here as in the previous case when V was equal to 2, viz. that P shall be moved to such a point, T or t, according as the epicycle is direct or retrograde, that the angle CMT, or CMt, shall be equal to $n \cdot$ MOA. That is, we have to find the value of the "angle of correction" PMT, or PMt, which is again designated by θ.

Taking PMT first: it is seen to be composed of the two angles PMC, CMT, of which PMC has just been shown to be $=\left(\frac{3x}{2y} - 1\right)$ OSM; and CMT has to be made $= n \cdot$ MOA, because CMT and MOA have to be described in equal times, P being supposed to move n times as fast as M.

It was also proved that, whatever be the length of OM, MOA $= \frac{\text{OSM}}{2}$. We have, therefore,

$$\text{PMT} = \text{CMT} + \text{PMC}$$
$$\text{or } \theta = n \cdot \frac{\text{OSM}}{2} + \left(\frac{3x}{2y} - 1\right) \text{OSM}. \quad . \quad . \quad . \quad . \quad (1)$$

Let OM be of the same value as before, = 0·01 inch,

$\therefore$ OSM = 0° 18′ as formerly determined.

And, by the nature of the case, if T be the point to

which P must be brought in order that it may arrive at C when M comes to A, we are dealing with a "direct" epicycle; and n is here

$$= \mathrm{V} + 1. \quad \left[\mathrm{V} \text{ being as usual } = \frac{3x}{y}\right].$$

Therefore the equation (1) becomes

$$\begin{aligned} \theta &= \left(\frac{3x}{y} + 1\right)\frac{\mathrm{OSM}}{2} + \frac{3x - 2y}{y} \times \frac{\mathrm{OSM}}{2} \\ &= \frac{y + 3x + 3x - 2y}{y} \times 9' \\ &= \frac{6x - y}{y} \times 9'. \end{aligned}$$

This result, which might be stated numerically for any particular values of x and y, expresses the angle through which the Frame has to be moved so that it may recover the position lost by a radial adjustment of one division of the Flange. But what is required is not the angle of correction *at the Frame*, but *at the tangent wheel*, which latter transmits to the former V times the amount of rotation it may itself receive.

Let C represent the required compensation, expressed in divisions of the micrometer screw, each of which has been shown to be equivalent to $0° \; 4'\cdot5$. Then, for all cases, the compensation will be $\frac{\theta}{\mathrm{V} \times 4\cdot5}$; and, in the present instance, (the loops being *internal*),

$$\begin{aligned} \mathrm{C} &= \frac{6x - y}{y} \times \frac{9}{4\cdot5} \times \frac{y}{3x} \\ &= \frac{2}{3} \times \frac{6x - y}{x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2). \end{aligned}$$

Next for the angle P M t: with the same arrangement of the instrument in other respects, let the

second "carrier" be withdrawn from the train. The epicycle will then be "retrograde," and $n = \text{V} - 1$.

P will be left by the same movement of the Flange in excentricity, at the same place as before (fig. 58); but the angle of correction P M t must now be above M C, in order that P, moving downwards from t after having been brought in correction to that point, may cross the datum line, B O A produced, at the moment when M is passing A. The Frame will evidently have to be turned in the opposite sense, by way of correction, from that which was found necessary in the last case; but, as there is now an axis less in the whole train, the instrument provides spontaneously for the alteration; and, whether the loops be external or internal, the compensation will be rightly transmitted by turning the micrometer screw forwards, i.e. in the order of its graduations.

P M t is therefore the difference of the two angles C M t, P M C; of which P M C, as in the last case, is $= \left(\frac{3x}{2y} - 1\right)$ O S M; and C M t is again $= n \cdot$ M O A, $= n \cdot \frac{\text{O S M}}{2}$, where n is now $= \text{V} - 1, = \frac{3x}{y} - 1$. We have therefore,

$$\text{P M } t = \text{C M } t - \text{P M C}$$

$$\text{or, } \theta = \left(\frac{3x}{y} - 1\right)\frac{\text{O S M}}{2} - \left(\frac{3x}{2y} - 1\right)\text{O S M} \quad . \quad . \quad (3)$$

$$= \frac{3x - y - 3x + 2y}{y} \times 9'$$

$$= 9'.$$

Reducing this to the corresponding value at the tangent wheel, we still have, for all cases, $\frac{\theta}{\text{V} \times 4{\cdot}5}$

as the compensation required; and using **C** in another type—which will be found a convenient distinction when the two kinds of compensation are in use for the same occasion, we have (the loops being *external*),

$$\mathbf{C} = \frac{9}{4 \cdot 5} \times \frac{y}{3x}$$

$$= \frac{2}{3} \cdot \frac{y}{x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

(ii) [See page 65.] If the experiment for comparing the relative directions of motion of Flange and Frame during the excentric adjustment of the former, be made with one of the pairs of change wheels which give for V a less value than 2, and therefore for the "latter part of the train" a *less value than unity*; it will be found that these directions of motion are now identical. The Frame, however, now revolves so slowly, that to be able to recognize its direction at all, it will be desirable to move the Flange through as large an arc as possible by temporarily removing its impelling screw, as was formerly recommended for a similar purpose.

It is at first somewhat perplexing to discover that a change of velocity can also entail a change in direction of motion. When all the axes of a train are fixed, this is out of the question; but it may occur with an "epicyclic train," as is explained by Professor Willis in the chapter upon that subject of his work already so often quoted. The Epicycloidal Cutting Frame forms indeed a complete illustration of "Ferguson's Paradox" there described.

(1.) If the value of the short train from the stud S to M on the Frame axis be *less* than unity—as in the case now supposed—the Frame revolves in the same direction as the Flange, (while the excentricity of the latter

is being altered, and the former part of the train is motionless).

(2.) If that value be *equal* to unity, as in the case of the ellipse, there is under the same circumstances no absolute revolution of the Frame whatever: and,

(3.) If that value be *greater* than unity, as in the case last discussed, the Frame and Flange revolve in opposite directions.

Whenever therefore it happens that V is less than 2, P will be brought to some point *below* M C (fig. 58) by the radial action of the Flange, and (the two angles P M C, O S M, being now of the same kind) the expression $PMC = \left(1 - \frac{3x}{2y}\right) OSM$ will stand without change of sign. The angle of correction will now, however, be evidently equal to the difference of the two angles P M C, C M T, instead of to their sum, when the epicycle is direct; and to the sum of the two angles P M C, P M *t*, instead of to their difference, when the epicycle is retrograde; and the general expression will remain unaltered.

Case I. (page 59) is evidently included in section (i) of Case II. For, when V = 2, the angle P M C does not occur, P always remaining at C; and the equations (1) and (3) each become reduced to $\theta = n \cdot \frac{OSM}{2}$, giving the two results stated at the top of page 64.

On the whole, therefore, when x and y are the two change wheels employed, x being that which is first placed on the removeable arbor, the corrections to be made at the Tangent wheel, for each division of excentricity added to the Flange, are to be applied in the same direction, (that in which the graduations are numbered) whether the loops be internal or external,

and are of the following values, expressed in divisions of the micrometer screw.

For *internal* loops,

$$C = \frac{2}{3} \times \frac{6x - y}{x}, = 4 - \frac{2y}{3x},$$

$$= 4 - \frac{2}{V}$$

For *external* loops, including the ellipse and straight line,

$$c = \frac{2}{3} \times \frac{y}{x}$$

$$= \frac{2}{V}$$

From these equations the proper corrections may be obtained for all available numbers of x and y.

The symmetry of the curve *per se* is never affected by any adjustment of the Tangent wheel: all curves retain under all circumstances their proper respective proportions. The disturbance, which it is the object of the present chapter to ascertain and to correct, becomes apparent solely by the position which the curve occupies (on the surface where it is traced) with respect to any datum line, real or imaginary.

NOTE.—If the value, 0° 8′ 36″, obtained on page 63, were adopted instead of its approximation 0° 9′, the Tangent wheel should contain 5023, or say 5000, equal parts instead of 4800. And therefore, if this wheel had 100 teeth instead of 96, the correction would be theoretically more exact. But the number 96 possesses more, and more convenient, factors than 100; and it is not probable that any *attainable* accuracy has been sacrificed by preferring the former.

CHAPTER V.

EXAMPLES OF THIS CORRECTION.

If it be simply required to place the figure vertically, then, after the Flange and Frame have been placed at right angles to one another, while the latter is also perpendicular to the lathe bearers and the former is strictly central, it is only needful to apply to the micrometer screw as many times C, or **c**, divisions as there are hundredths of an inch (i.e. divisions) in the excentricity about to be given to the Flange. And if it be then desired to add other curves of the same formation and direction, but with different excentricities, their parallelism to the first will be attained by adding or subtracting C, or **c**, divisions at the Tangent wheel micrometer for each division by which the excentricity of the Flange is increased or diminished.

Supposing for example that the wheels $x = 40$, $y = 42$, are adopted, producing a figure of 20 loops; and that the curve is internal: we have

$$\begin{aligned} C &= \frac{2}{3} \times \frac{240 - 42}{40} \\ &= \tfrac{2}{3} \times \tfrac{198}{40} \\ &= \tfrac{33}{10} = 3{\cdot}\dot{3} \end{aligned}$$

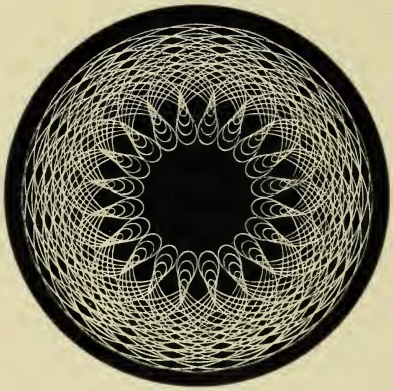
Similarly, if the loops be external,

$$\begin{aligned} \mathbf{c} &= \tfrac{2}{3} \times \tfrac{42}{40} \\ &= \tfrac{14}{20} = 0{\cdot}7 \end{aligned}$$

The next two designs are derived from these values of *x* and *y*, and are corrected in accordance with the above calculations.

In fig. 60, *b* was = 21 throughout, and *a* was increased by 5 divisions at a time, from 50 to 70, for the five curves respectively.

Fig. 60.



After the adjustment for "initial position" (i.e. Flange and Frame perpendicular to one another, and Frame perpendicular to lathe bearers) had been satisfactorily accomplished, 3 turns and 15 divisions (= 3·3 × 50) were moved at the micrometer of the tangent wheel from its adopted zero point. This corrected the position of the first curve; and, for each of the four remaining, the added correction at the tangent wheel was 16·5 divisions (=3·3 × 5).

Fig. 61 is the converse of its predecessor: *b* was = 30 throughout, and the values of *a* were from 40 to 60 inclusive, increasing by intervals of 5 divisions. The correction for the first, or inmost, curve was 28 divisions (= 40 × 0·7) at the tangent wheel; and further

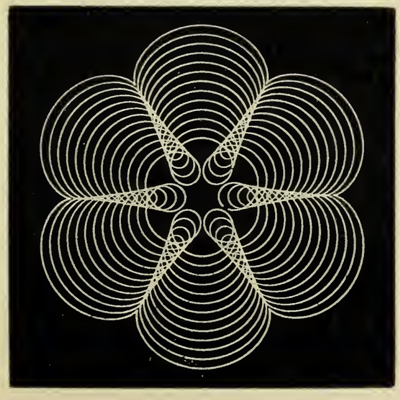
quantities of 3·5 divisions (= 5 × 0·7) were moved in correction for each of the succeeding lines. As a matter of effect, the curve exterior to the cusp might

Fig. 61.

have been omitted with advantage: it serves, however, to render the success of the correction more evident.

The following is a corrected form of fig. 53, the adjustments being the same here as in that figure, as regards the change wheels and the excentricities of Flange and Frame.

Fig. 62.



To bring all the curves into a similar position, the compensation, as given by the formula for external loops, was applied at the rate of 3·66 divisions of the tangent wheel micrometer for each division of the Flange. For the first curve, whose value for a was 20, the correction was ($20 \times 3·66 =$) 1 turn, 23·3 divisions: and for the last, where $a = 80$, the correction was 5 turns, 43 divisions. The intervening curves, distant 5 divisions of the Flange from one another, were each corrected by ($5 \times 3·66 =$) 18·3 divisions of the tangent screw.

For loops from 2 to 16 in number, of those within the range of the Epicycloidal Cutting Frame, the necessary corrections have been calculated, from the above formulæ, for both directions of the curve, and will be found in Table IV. They are expressed in vulgar fractions as well as in decimals, as the former, besides being often more accurate, are, for some increments in the values of a, also more convenient. The treatment of the compensation may frequently be facilitated by some slight modification of intended consecutive values for a; such values being adopted as will require whole numbers of divisions at the micrometer screw, or fractions of a division that are easily estimated, as $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{1}{3}$. For, in the vulgar fraction which is found to be the equivalent of C, or $\mathbf{c}$, for any given pair of wheels, the *numerator* indicates the number of divisions at the Tangent wheel which will compensate for an excentricity of the Flange of as many divisions as are expressed by the *denominator*. For instance, in the example just given.

20 loops, C	$= 16\frac{1}{2}$	T.W. for	5 Fl.	$\mathbf{c}$	$= 7$	T.W. for	5 Fl.
	$= 8\frac{1}{4}$	,,	$2\frac{1}{2}$,,		$= 3\frac{1}{2}$	,,	$2\frac{1}{2}$,,

and for

7 loops,	C =	12	T.W. for	$3\frac{1}{2}$ Fl.	**c** =	2	T.W. for	$3\frac{1}{2}$ Fl.
12 ,,	C =	$9\frac{1}{2}$	,,	3 ,,	**c** =	$2\frac{1}{2}$	,,	3 ,,
	=	$4\frac{3}{4}$	,,	$1\frac{1}{2}$,,	=	$1\frac{1}{4}$	,,	$1\frac{1}{2}$,,
30 ,,	C =	16	,,	$4\frac{1}{2}$,,	**c** =	2	,,	$4\frac{1}{2}$,,
	=	8	,,	$2\frac{1}{4}$,,	=	1	,,	$2\frac{1}{4}$,,

A convenient relation of this kind can be framed in nearly every case: and the consecutive values of a can generally be arranged accordingly. When that cannot be done, owing to some special values being required for a, the compensation will have to be expressed decimally in terms of one division of the Flange.

It may be sometimes useful by way of testing the accuracy of a calculation, to remember that the sum of the two kinds of compensation, for any given values of x and y, is in all cases *equal to the number* 4. This will be at once apparent by the addition of the algebraical quantities representing C and **c** respectively. Whenever, therefore, both kinds of compensation are likely to be required, it will be convenient to calculate first the simpler form for external loops, and then to use (4 − **c**) as the value for C.

And it has been pointed out by Mr. Pomeroy—whose assistance in revising these Notes has been of much value—that as regards the compensation for *external* loops, two divisions should be moved at the tangent screw for as many divisions on the Flange as are equal to the number of loops in the curve. For example, the compensation

for 2 loops (external) is 2 at the Tangent Screw for 2 on the Flange.

,, 4 ,, ,, 2 at the Tangent Screw for 4 on the Flange.

for 6 loops (external) is 2 at the Tangent Screw for 6 on the Flange.

And for 7 loops (a figure resulting from the duplication of $3\frac{1}{2}$ loops) the compensation is 2 at the tangent screw for $3\frac{1}{2}$ on the Flange. In the same manner, for 16 loops ($x = 32, y = 30$), based upon $3\frac{1}{5}$ loops, the compensation is 2 at the tangent screw for $3\frac{1}{5}$ on the Flange. This may be expressed as 1 at the tangent screw for $\frac{16}{10}$ on the Flange, or 1 on the Flange for $\frac{5}{8}$ at the tan-

TABLE IV.

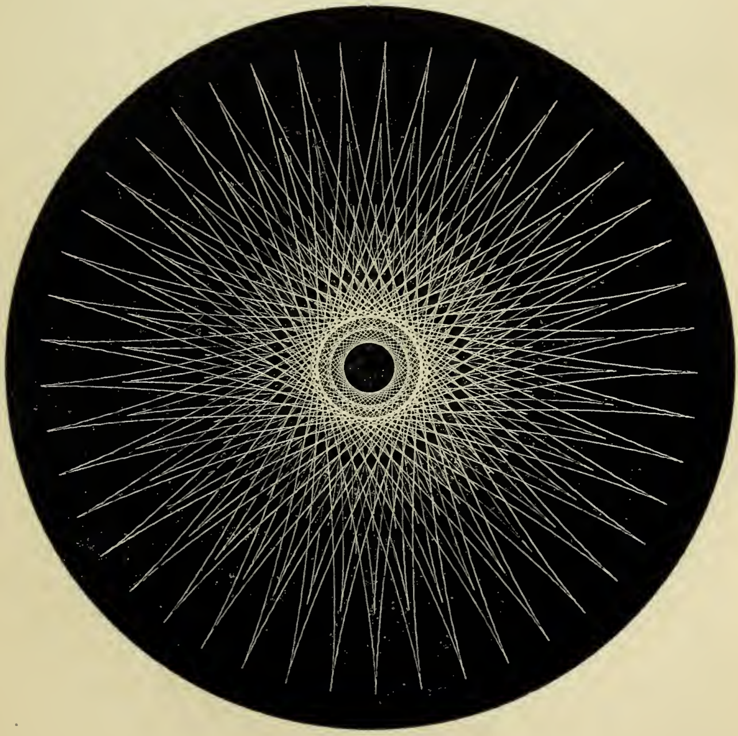
Loops	x	y	V	Divisions to be moved at Micrometer Screw of Tangent Wheel for one Division of Excentricity on the Flange	
				External	Internal
2	32	48	2	1·	3·
3	48	48	3	$\frac{2}{3} = 0{\cdot}6\dot{6}$	$3\frac{1}{3} = 3{\cdot}3\dot{3}$
4	48	36	4	$\frac{1}{2} = 0{\cdot}5$	$3\frac{1}{2} = 3{\cdot}5$
5	60	36	5	$\frac{2}{5} = 0{\cdot}4$	$3\frac{3}{5} = 3{\cdot}6$
"	40	48	2·5	$\frac{4}{5} = 0{\cdot}8$	$3\frac{1}{5} = 3{\cdot}2$
6	60	30	6	$\frac{1}{3} = 0{\cdot}3\dot{3}$	$3\frac{2}{3} = 3{\cdot}6\dot{6}$
7	42	36	3·5	$\frac{4}{7} = 0{\cdot}57$	$3\frac{3}{7} = 3{\cdot}43$
8	32	36	2·6$\dot{6}$	$\frac{3}{4} = 0{\cdot}75$	$3\frac{1}{4} = 3{\cdot}25$
9	60	40	4·5	$\frac{4}{9} = 0{\cdot}4\dot{4}$	$3\frac{5}{9} = 3{\cdot}5\dot{5}$
"	36	48	2·25	$\frac{8}{9} = 0{\cdot}88$	$3\frac{1}{9} = 3{\cdot}1\dot{1}$
10	40	36	3·3$\dot{3}$	$\frac{3}{5} = 0{\cdot}6$	$3\frac{2}{5} = 3{\cdot}4$
11	44	36	3·6$\dot{6}$	$\frac{6}{11} = 0{\cdot}54$	$3\frac{5}{11} = 3{\cdot}46$
"	44	48	2·75	$\frac{8}{11} = 0{\cdot}73$	$3\frac{3}{11} = 3{\cdot}27$
"	44	60	2·2	$\frac{10}{11} = 0{\cdot}91$	$3\frac{1}{11} = 3{\cdot}09$
12	32	40	2·4	$\frac{5}{6} = 0{\cdot}8\dot{3}$	$3\frac{1}{6} = 3{\cdot}16$
15	40	32	3·75	$\frac{8}{15} = 0{\cdot}53$	$3\frac{7}{15} = 3{\cdot}47$
"	30	42	2·14	$\frac{14}{15} = 0{\cdot}93$	$3\frac{1}{15} = 3{\cdot}07$
"	30	48	1·87	$\frac{16}{15} = 1{\cdot}07$	$2\frac{14}{15} = 2{\cdot}93$
16	32	30	3·2	$\frac{5}{8} = 0{\cdot}62$	$3\frac{3}{8} = 3{\cdot}38$
"	32	42	2·28	$\frac{7}{8} = 0{\cdot}87$	$3\frac{1}{8} = 3{\cdot}12$

gent screw, which agrees with the correction as given in Table IV. for this variety of the 16-looped figure.

When the excentricity of the Flange is increased or diminished by a quantity which is a multiple of the number of loops produced, this rule may be of much service; but it is applicable to *external* loops only. For *internal* loops there does not appear to be any equally simple relation subsisting between the formation of the curve and the correction at the tangent screw.

The proportion thus shown to exist between the number of loops in the curve, and the compensation which it requires, is the natural interpretation of the formula

Fig. 63.



$\mathbf{c} = \frac{2}{V}$ given at the conclusion of the last chapter; and on some occasions this simpler expression may be the more convenient.

The table of compensations might have been extended further, but corrections for the higher loops are seldom necessary; and when required can be readily obtained from the same formula.

For instance, the specimen (fig. 63), on the last page, of 45 loops outwards ($x = 30$, $y = 38$, $V = 2{\cdot}37$) consists of three curves only, all brought to the cusped condition $\left(b = \frac{a}{n}\right)$, and the compensation was

$$\frac{2}{3} \cdot \frac{y}{x} = \frac{38}{45} = 0{\cdot}84.$$

The adjustments stood thus, **c** being reckoned for each curve from the zero point of the tangent wheel.

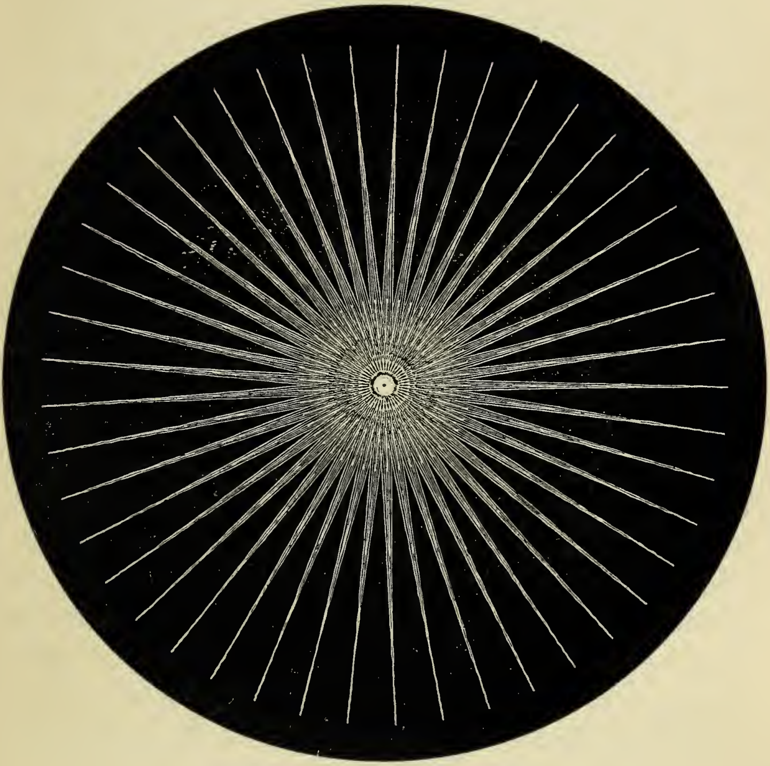
$a =$	50,	$b =$ 36·5,	**c** = 42·2
	75,	54·7,	63·2
	100,	73,	84·4.

As an extreme instance, and one of the class where V is less than 2, fig. 64 is a good example, though not very suitable to purposes of ornamentation.

The effect of the engraving is injured to some extent by a certain tremulousness apparent in the outside curve, caused partly by the abrupt changes in the direction of motion when the excentricities are so considerable, and partly by a want of sufficient care in increasing the penetration of the tool when such large radii are employed; but, principally, from there having been too much "play" between the change wheels and those with which they were connected. A magnifying glass will show that there are four curves, all cusped,

as being the condition in which a want of parallelism to those adjacent will be the most perceptible.

Fig. 64.



$x = 30$, $y = 46$, $V = \frac{45}{23}$, $= 1{\cdot}95$, one carrier, $n = \frac{22}{23} = \frac{a}{b}$ (fig. 64).

$$\mathbf{c} = \frac{2}{3} \cdot \frac{y}{x} = \frac{2}{3} \cdot \frac{46}{30} = \frac{46}{45} = 1{\cdot}02$$

$$\begin{array}{llllll} a & = & 22, & 44, & 66, & 88 \\ b & = & 23, & 46, & 69, & 92 \\ \mathbf{c} & = & 0, & 22{\cdot}4, & 44{\cdot}9, & 67{\cdot}3 \end{array}$$

Sufficient proof has probably been now given that the formulæ, deduced theoretically for obtaining the correction of the inequality caused by the radial excentric action of the Flange, stand the test of experiment satisfactorily. We are now, therefore, in a position to copy, or to modify, any design within the limits of the instrument; or to adapt to any specified extent of surface, any desired arrangement of curves.

Taking for instance the two examples at the foot of the first page of the sheet of diagrams, with which the reader will probably be familiar, published by Messrs. Holtzapffel & Co., "illustrating the Epicycloidal Cutting Frame," we observe that the first (at the left hand bottom corner) is evidently composed of 9 loops inwards, and by following with a needle point the course of any one of the lines, it is not difficult to decide that the nine loops are formed by twice $4\frac{1}{2}$ loops, and not by four times $2\frac{1}{4}$ loops; these being the two varieties attainable. The outside curve is cusped; and, by careful measurement with fine compasses and a diagonally divided scale, the diameters of the imaginary circumscribed and inscribed circles—i.e. the apocentral and pericentral circles—appear to be 1·52 and 1·08 inches respectively.

Now it was shown, page 18, that (*a*) the excentricity of the Flange is in all cases equal to one fourth of the sum of the diameters of the apocentral and pericentral circles; and that (*b*) the excentricity of the Frame is equal to one fourth of their difference: we have, therefore, in the present instance,

$$a = \frac{152 + 108}{4} = 65 \text{ divisions}$$

$$b = \frac{152 - 108}{4} = 11 \quad \text{,,}$$

Similar measurements for the inmost looped curve give 0·8 inch and 0·32 inch as the diameters of its apocentral and pericentral circles,

$$\text{whence } a = \frac{80 + 32}{4} = 28 \text{ divisions}$$

$$b = \frac{80 - 32}{4} = 12 \quad \text{,,}$$

There are thirteen curves altogether, occupying a width of (65 − 28 =) 37 divisions; but it is more probable that 36 divisions, giving 3 to each of the 12 spaces, is the true width. It is also obvious that the pattern was completed with one value for (b); increase of excentricity being given to (a) only; and this estimated value of (b) comes out 11 in one instance and 12 in the other. This discrepancy may arise from the difficulty of measuring by compasses to 0·01 inch, enhanced in this case by the fact that no two loops of the same curve lie upon the same diameter; as well as from a possible irregular contraction of the paper after printing. The number 11 seems to be nearer the mark than 12 for the excentricity which has been given to the Frame for this design; but, checking this by the fact that the outside curve must be cusped,

$$\text{we have } b = \frac{a}{n}, \text{ where } n = 5\tfrac{1}{2};$$

$$\text{and, if } b = 11, \qquad a = 60{\cdot}5;$$

rather too little:

$$\text{while, if } b = 12, \qquad a = 66.$$

It was therefore decided to enlarge the pattern slightly, to adopt 12 divisions for (b); and to describe fourteen curves in all instead of thirteen. The com-

pensation required, as stated in Table IV., is 3·55 divisions at the screw of the tangent wheel for one on the Flange: this is equal to 10·65 for three on the Flange; and the successive adjustments of the latter by three divisions at a time from 27 to 66 inclusive, were corrected by that amount. The result appears at fig. 65.

Fig. 65.

Of course it is not possible to estimate hundredths, or even tenths, of a division on the micrometer screw head; but, in tabulating beforehand (as it is prudent to do) the values about to be used for a, b, and C, two decimal places should be used in order to ensure the correctness of the first.

$x = 60$, $y = 40$, two carriers, V = 4·5, loops (9) internal, fig. 65.

a = 27,	b = 12,	C = 1 .	45·85
30,	,,	2 .	6·5
33,	,,	,,	17·15
36,	,,	,,	27·8
39,	,,	,,	38·45
42,	,,	,,	49·1
45,	,,	3 ·	9·75

$a = 48$,	$b = 12$,	$C = 3 \cdot 20{\cdot}5$
51,	,,	,, 31·15
54,	,,	,, 41·8
57,	,,	4 · 2·45
60,	,,	,, 13·1
63,	,,	,, 23·75
66,	,,	,, 34·4

The quantities in column C were increased by the reading at which the tangent wheel stood when its zero point had been ascertained. There is but a slight probability, with any arrangement of wheels, that what has been termed the "initial position" can be attained, and the tangent wheel be brought simultaneously to the zero of its graduations. Nor is it desirable that it should be so; for the wheel and screw will wear much more equably by continually changing their points of contact, to all parts of the circumference of the wheel. In making this preliminary adjustment, the Frame should be brought to the vertical position by being moved in the direction in which it is about to travel. Consequently the lines indicating the horizontality of the Flange should be brought to coincide by moving the pulley upwards or downwards, according as one or both "carriers" may be connected with the train. It is absolutely essential that the Flange have no excentricity while this adjustment is in progress. The greater the difference in speed between the Flange and Frame, i.e., the higher the value of V, the greater is the effect of the tangent wheel in altering the inclination of the Frame. When for example, $x = 60$, $y = 30$, less than a division of the micrometer screw makes a very perceptible difference in the inclination of the Frame; but when $V = 2$ or thereabouts, the effect of

the tangent wheel in this respect is much less, and the adjustment for verticality is therefore both more easy and more certain.

The diagram (in the sheet of illustrations referred to) adjoining that which, with some slight variation, has just been copied, consists of two groups of curves, each with 15 loops, one set external, the other the reverse. The same difficulty of measurement occurs here as with the former figure; because the loops being uneven in number are not placed diametrically.

But, beginning with the outer group, the dimensions appear to be as follows:—

outside curve, exterior diameter,		150
,, interior	,,	102

$$\therefore a = \frac{150 + 102}{4} = 63 : \text{ and } b = \frac{150 - 102}{4} = 12.$$

inside curve, exterior diameter,		135
,, interior	,,	90

$$\therefore a = \frac{135 + 90}{4} = 56\tfrac{1}{4} : \text{ and } b = \frac{135 - 90}{4} = 11\tfrac{1}{4}.$$

The loops prove on examination to be fifteen in number; and a tracing point passed over the course of the curve indicates that $3\frac{3}{4}$ loops occupy the circumference of the circle. Therefore, on reference to the tables, we see that 15 loops, where $V = 3{\cdot}75$, are produced by the change wheels $x = 60$, $y = 48$; or $x = 40$, $y = 32$.

If it be desired that the three curves forming this group should all be cusped, which is not quite the case in the original, we must have for each,

$$\frac{a}{b} = n = 1 + \mathrm{V} = 1 + 3\tfrac{3}{4} = \tfrac{19}{4}.$$

The nearest exact ratio corresponding to the dimensions found by measurement is $\frac{57}{12}$; and the following were the values adopted :—

$$\left.\begin{array}{ll} a = & 57, \quad 59{\cdot}5, \quad 62 \\ b = & 12, \quad 12{\cdot}5, \quad 13 \\ \mathrm{C} = & 3.47{\cdot}7, \ 4.\,6{\cdot}4, \ 4.\,15 \end{array}\right\} \begin{array}{l}\text{fig. 66,} \\ \text{internal cusps.}\end{array}$$

The compensation, expressed in turns and divisions of the tangent screw, is, as usual, the product of a into the tabular correction [C = 3·47].

For the external loops forming the centre of the design, the dimensions are :

outside curve, exterior diameter, 85
,, interior ,, 30

$$\therefore a = \frac{85 + 30}{4} = 28\tfrac{3}{4} : \text{ and } b = \frac{85 - 30}{4} = 13\tfrac{3}{4}.$$

inside curve, exterior diameter, 75
,, interior ,, 17

$$\therefore a = \frac{75 + 17}{4} = 23 : \text{ and } b = \frac{75 - 17}{4} = 14\tfrac{1}{2}.$$

These calculations seem to indicate that the value for (b) was 14 in each of the three curves, and that the values of (a) were 23, 26, and 29. They were employed accordingly, and carefully corrected for symmetry of position. (C = 0·53.)

$$\left.\begin{array}{ll} a = & 23, \quad 26, \quad 29 \\ b = & 14, \quad 14, \quad 14 \\ \mathrm{C} = & 12{\cdot}2, \ 13{\cdot}8, \ 15{\cdot}4 \end{array}\right\} \text{fig. 66. (centre.)}$$

In cases of this kind, where external and internal loops of the same number are brought so pointedly into juxtaposition, much care is required in defining the zero position of Flange and Frame for both directions of motion in their turn, before using the value for correction at the tangent wheel. When the needful precautions in this respect are attended to, the change wheels and the carriers may be varied at pleasure, without interfering with the depth of the cut, or the symmetry of the figure.

Fig. 66.

The addition of the border of 45 loops ($x = 60$, $y = 32$, $a = 97$, $b = 21$) cannot be said to be an improvement in its present form. The curves might have been repeated once, or oftener, with diminishing values for (b): or one figure of 90 loops might have been advantageously substituted. In the latter case, the most suitable wheels would have been $x = 60$, $y = 34$,

whence V = 5·29, and the values a = 94, b = 18, would have given a near approach to the cusped condition, and a pericentral boundary identical with the above. In calculating the proportions of a figure to be placed concentrically as closely as possible to the boundary of one already traced in the centre of the design, without encroaching upon the latter, we must have $(a - b)$ for the new curve, greater than $(a + b)$ for the old one. The outside 15-looped, or rather cusped, curve (fig. 66) had $a + b = 75$; and for the border $(a - b)$ was taken at 76, thus leaving an interval between the two figures, of a hundredth of an inch, all round.

The reproduction of the designs, selected for that purpose, has been discussed with perhaps a tedious minuteness: to the following example, suggested by one of the figures on the back of the stereotyped catalogue of Messrs. Holtzapffel & Co., the adjustments are appended without further remark.

Fig. 67.

For the cusped centre, $x = 32$, $y = 36$, loops (8) external, $\mathbf{c}$ = 0·75.

$a =$ 5, 7·5, 10, 12·5, 15
$b =$ 3, 4·5, 6, 7·5, 9
c = 3·7, 5·6, 7·5, 9·4, 10·3

For the external loops, $x = 48$, $y = 36$, loops (4) external, **c** = 0·5.

$a =$ 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40.
$b =$ 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35.
c = 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20.

For the internal loops, $x = 48$, $y = 36$, loops (4) internal, C = 3·5.

$a =$ 50, 52, 54, 56, 58, 60, 62
$b =$ 25, 23, 21, 19, 17, 15, 13
C = 3.25, 3.32, 3.39, 3.47, 4. 3, 4.10, 4.17.

The analysis of a given design will frequently be not quite so simple. Curves may have the same number of loops, and yet, though bounded by the same imaginary circles, may have widely different aspects; and, owing to their consequently different values for V, will require different change wheels for their production. They may also be associated in such a manner (as will be described subsequently) as to present the appearance of a single continuous curve of a higher number of loops. There can be no doubt, however, as to the dimensions of the figure which may be offered for imitation, and therefore none as to the requisite excentricities of Flange and Frame. These quantities having been ascertained by the rule stated above: viz., for (a) taking half the sum, and for (b) half the difference, of the radii of the circumscribed and inscribed circles: the adjustments may be effected accordingly. Various change wheels can then be tried in succession

in such pairs as will produce the given loops, singly or by composition; and the desired curve will be discovered, provided it be within the limits of the instrument. With regard to "printed patterns" in general, some valuable hints are given in a work on "Eccentric Turning by an Amateur" (Engleheart); the specimens of which are executed with marvellous accuracy, and exquisitely printed.

It may be convenient to repeat, in concluding this chapter, that *a* and *b* represent the excentricities of Flange and Frame, stated in their respective divisions; —i.e. in hundredths of an inch: and that C (or **C**, as the case may be) denotes the number of divisions through which the Tangent screw should be moved to compensate the obliquity introduced by the addition of one division to the Flange.

In the *tabulated adjustments*, C (or **C**) indicates the number of divisions,—and sometimes of whole turns and divisions,—which should be the reading of the Tangent screw, for each consecutive value of *a*, reckoned from the zero point of the "initial position."

CHAPTER VI.

INTERPOLATION OF CURVES, AND THEIR SPIRAL ARRANGEMENT.

HITHERTO the circular movement at the back of the instrument, described as consisting of a tangent wheel and micrometer screw, has been referred to solely as a means of restoring to a group of curves an identity of position which had been disturbed by the radiation of the Flange. It possesses, however, another very useful function; that of distributing, round a common centre, copies of a given curve; preserving any desired distance, equal or unequal, between the consecutive branches. A repetition of curves involving high numbers of loops can however only tend, except in unusually large diameters, to overcrowding and indistinctness: it is in the duplication of simple "consecutive" curves chiefly, that this property is sometimes advantageous. In cases of this kind, that is to say when the loops are 3, 4, 5, or 6 in number, it may afford occasional variety to repeat the curve, at such intervals as will complete the figure, instead of adopting a "circulating" curve which would produce a similar figure by one adjustment.

It is obvious for example that a figure of 24 loops, internal or external, may be built up from one of 6 loops only, by repeating thrice the single curve first described. A similar figure of an equal number of loops might be constructed by sufficient repetitions of

the 3, or 4, looped figure; and also by doubling the circulating curve with twelve loops, or trebling that with eight.

By this method of interpolation, the 3 looped figure, internal or external, may be converted into one with 6, 9, 12, 15, 18, 21, &c., loops ;—

the 4 looped figure into one with 8, 12, 16, 20, 24, &c., loops
the 5 looped figure into one with 10, 15, 20, 25, 30, &c., loops
the 6 looped figure into one with 12, 18, 24, 30, 36, &c., loops

and the circular movement of the Tangent wheel, which, from its 4800 graduations, constitutes a division plate of no ordinary resources, ensures the accurate distribution of the repeated curves.

Special numbers of loops not to be found in the Tables may be obtained in this manner, for instance 14 and 25. And, where the Tables give for any number, only a limited selection of values for V, upon which, as determining the value of n, it has been seen that the general character of the curve so largely depends, this system of repetition will often allow more choice in this respect. Thus the highest value for V when the loops are 48, is stated in Table II. to be 2·82; but if it be desired that the figure should be compressed into an annular space of less width, still preserving the number of loops and their general appearance, this can be done by adopting 4 or 6 as the value for V, and completing the figure by repetitions of that with 4 or 6 consecutive loops.

The following diagrams illustrate the composition of a 24 looped figure from the several elements suggested

above. In *each* case the angular interval through which the tangent wheel is moved will be the 24th part of the circumference; or $\frac{\text{96 teeth}}{\text{24 equal parts}} = 4$ turns of the tangent screw between the adjacent loops. But the number of interpolations required between the consecutive loops of the original curve will be less as those loops are more numerous.

Fig. 68.

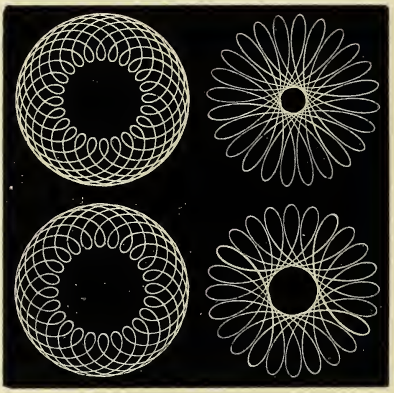


Fig. 68. Two upper figures.

V = 3, loops (3) internal, $x = 48$, $y = 48$, $a = 33$, $b = 12$.
V = 3, loops (3) external, $x = 48$, $y = 48$, $a = 26$, $b = 19$.

Two lower figures.

V = 4, loops (4) internal, $x = 48$, $y = 36$, $a = 29$, $b = 16$.
V = 4, loops (4) external, $x = 48$, $y = 36$, $a = 34$, $b = 11$.

In the last figure of this diagram, one of the 4 looped curves is cut rather more deeply than the rest, to show their individual character and the number of repetitions.

Fig. 69.

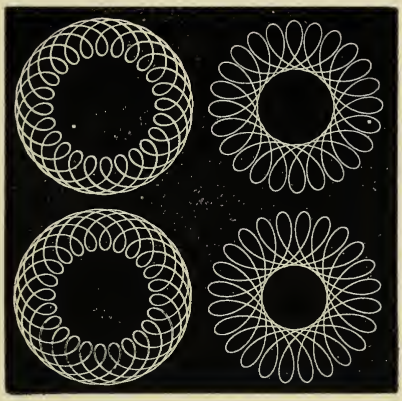


Fig. 69. Two upper figures.

V = 6, loops (6) internal, $x = 60$, $y = 30$,
$a = 36$, $b = 10$.

V = 6, loops (6) external, $x = 60$, $y = 30$,
$a = 32{\cdot}5$, $b = 12{\cdot}5$.

Two lower figures.

V = 4·8, loops (24) internal, $x = 48$, $y = 30$,
$a = 35$, $b = 10$.

V = 4·8, loops (24) external, $x = 48$, $y = 30$,
$a = 31$, $b = 14$.

Fig. 70. Two upper figures.

V = 2·66, loops (8) external, $x = 32$, $y = 36$,
$a = 24{\cdot}5$, $b = 20{\cdot}5$.

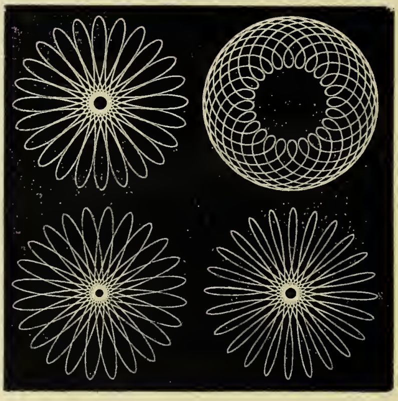
V = 2·66, loops (8) internal, $x = 32$, $y = 36$,
$a = 32{\cdot}5$, $b = 12{\cdot}5$.

Two lower figures.

V = 2·17, loops (24) external, $x = 32$, $y = 44$, $a = 21·5$, $b = 23·5$.

V = 2·4, loops (12) external, $x = 32$, $y = 40$, $a = 24$, $b = 21$.

Fig. 70.



When the loops are internal, the same general resemblance occurs here as in previous diagrams; and there is hardly the difference which might have been expected between the external forms of the three compounded figures and the single circulating curve. To some extent this difference depends upon the comparative values employed for V. In fig. 69 the value for the circulating curve was 4·8, and the resulting curve, in both forms, is therefore intermediate in character between the two which originate from the 4 and 6 looped curves respectively. In fig. 70 the external form is given (at the left hand lower corner) of a 24 looped figure described with the lowest value for V, (2·17) with which it can be obtained. This curve, which is very distinct from that which corresponds to it in fig. 69, is of the class formerly noticed where n is

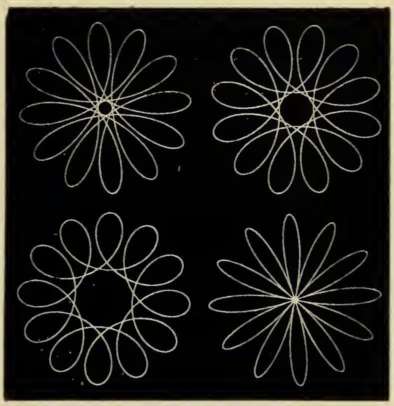
negative and nearly = 1; and to avoid the extreme narrowness of the loops, b was taken in excess of a, so that the loops intersect and pass beyond the centre.

Fig. 71 exhibits all the methods by which 12 equidistant loops can be described; and contrasts, rather more plainly than the preceding, the different effects which the same number of loops may produce—

(i) when described in a single curve, and

(ii) when constructed by different methods of interpolation.

Fig. 71.



The external forms only of each variety are given; for the internal forms are very similar to one another and to those which have already appeared.

Fig. 71. Two upper figures.

V = 3, loops (3) $x = 48$, $y = 48$, $a = 24{\cdot}5$, $b = 20{\cdot}5$.
V = 4, „ (4) $x = 48$, $y = 36$, $a = 27$, $b = 18$.

Two lower figures.

V = 6, loops (6) $x = 60$, $y = 30$, $a = 29{\cdot}5$, $b = 15{\cdot}5$.
V = 2·4, „ (12) $x = 32$, $y = 40$, $a = 22{\cdot}5$, $b = 22{\cdot}5$.

In determining the extent to which the tangent wheel must be moved in all these cases, it is only necessary to bear in mind that this angular interval corresponds to the number of loops in the complete figure. Thus, in the three compound figures of the last diagram, the complete number of loops is 12: therefore the tangent wheel was moved, in all three cases, through the twelfth part of its circumference (= 8 turns of the tangent screw), between every two adjacent curves. And this adjustment was required thrice for the 3 loops, twice for the 4, and only once for the 6. In the same manner, if a figure of 20 loops be compounded from one of 5,—which may be occasionally useful, as the Tables afford but one such figure, viz. when V = 2·86,—the tangent wheel adjustment will be $\frac{96}{20}$ = 4 turns and 40 divisions of the micrometer screw.

Pleasing effects may be had from a partial, or intermittent, system of interpolation; as shown, on rather a larger scale, in the two diagrams below.

Fig. 72.

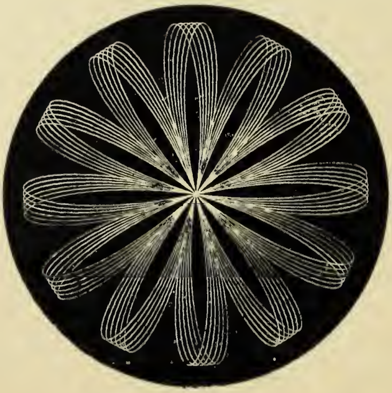


Fig. 73.

Fig. 72. $x = 32$, $y = 40$, loops (12) external, $a = b = 45$, six curves, tangent screw moved one turn between each.

Fig. 73. $x = 32$, $y = 40$, loops (12) internal, $a = 62$, $b = 28$, four* curves, tangent screw moved one turn between each.

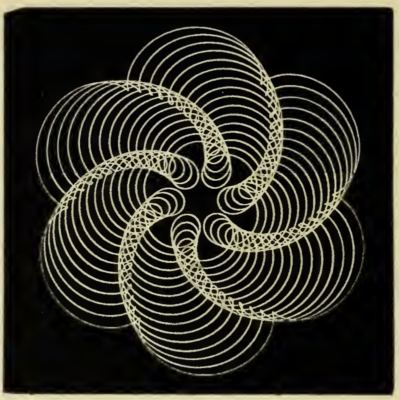
It is also possible to use the tangent wheel adjustment in aggravation, instead of in correction, of the disturbance produced by the radial action of the Flange; and also to apply the correction itself in excess, or in deficiency. The effect of these experiments will be to arrange the consecutive curves in spiral order: and the spirals may be either right- or left-handed, and of any desired pitch, whether uniform or variable.

Referring to fig. 53, it will be seen that the spiral of moderate inclination, there produced by the uncorrected adjustment of the Flange, might be varied in pitch as readily as it has been altogether neutralised in fig. 60. This has been accomplished in fig. 74, where the read-

* Five would make the figure more complete.

ing of the tangent screw was increased by 40 divisions between each curve; and the natural deviation caused by the uncorrected action of the Flange is consequently augmented. The curves themselves are more

Fig. 74.

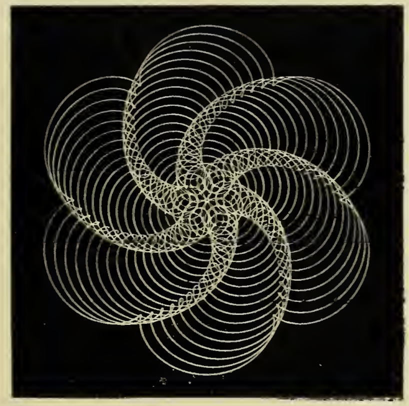


numerous, and more closely placed than in fig. 53, being described at intervals of 4 divisions of the Flange instead of 5; but *a* and *b* had the same values as in that figure.

If the spiral were required to be reversed, having the same pitch inclined in the opposite direction, it would be necessary to apply a two-fold correction; one to counteract the deviation of the instrument, the other to create the spiral: and the former would have to be applied *twice*; once to bring the curve into the vertical position, and once more as an equivalent to that natural deviation which produced the spiral in fig. 53, and contributed towards it in fig. 71. The tabular correction for 6 loops internal is $3\frac{2}{3}$, or $13\frac{1}{3}$ for 4 divisions of the Flange; therefore the spiral in fig. 74 may be said to have been influenced by ($40 + 13\frac{1}{3} =$) $53\frac{1}{3}$ divisions

of the tangent screw; and, to obtain an exactly reversed figure, we must add to this another $13\frac{1}{3}$, making $66\frac{2}{3}$ divisions, or just one turn and one-third of the tangent screw for the total amount which it should

Fig. 75.



receive between each curve to obtain the prescribed effect.

Fig. 75 was engraved in this manner, with the same values for a and b as previously: the series of curves being now continued to the centre to form a distinction between the two figures. It is more necessary, however, here, than in considering the generality of printed patterns, to recollect that the operation of printing is one of *inversion*; the upper part on the wood becomes the lower part on the paper, (unless a re-inversion should occur by the manner in which the block is placed in the press,) and the right and left similarly change sides. And it thus happens, as regards the actual appearance of the work in the lathe, that fig. 74 applies to the description of fig. 75, and *vice versâ.*

It will readily occur to the possessor of an ornamental lathe with its usual adjuncts, that if greater

variety be sought than is afforded by the Epicycloidal Cutting Frame in a central position, it may be obtained to any extent by using the Division Plate, the Excentric Chuck, and the graduations of the Slide Rest, singly or in combination. The tangent wheel may itself be used as a division plate, with the large selection of numbers of equal parts offered by 4,800, the sum of its divisions; and when the curves are placed excentrically, and the division plate of the lathe is therefore not available, their angular position can only be changed by having recourse to the tangent wheel. Numerous examples of these modes of treatment are unnecessary, but the two following diagrams may be useful in suggesting more ornamental effects obtained in a similar manner.

Fig. 76.

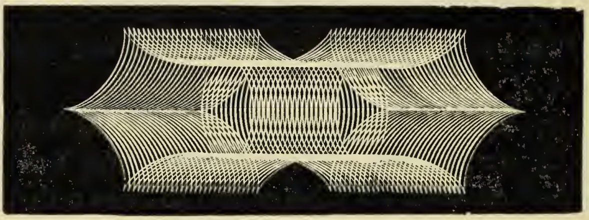
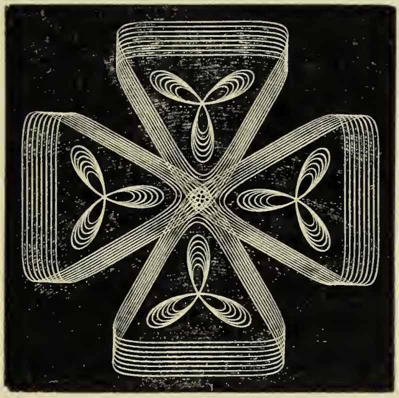


Fig. 76 is a series of curves with five cusps [$x = 60$, $y = 36$] placed 0·025 inches apart by the Slide Rest; the second portion of the series being described when the mandrel had been turned half round after the first was finished. Considering the figure as a polygon, (which might have been used with equal success, taking $a = 16\,b$, instead of $a = 4\,b$, as here,) it was necessary that one side should be vertical. Therefore, since the interior angle of a pentagon is 108°, one side will be brought into the required position by moving the tangent wheel through 18°, $= \frac{1}{20}$ of the circumference,

= 4 turns, 40 divisions of the micrometer screw, after the preliminary correction has been settled.

Fig. 77.



The Maltese cross in outline, fig. 77, which might be enriched in various ways, is derived from the polygonal form of the external three-looped figure: one side of the triangle was made vertical by adding to the tangent wheel 30°, = $\frac{1}{12}$ circumference, = 8 turns of the micrometer screw, after the position of the figure had been first corrected in the usual manner. The Slide Rest movement provided for the repetition of the parallel figures, and the Division Plate, by being moved continuously through one-fourth of any of its circles, completed their arrangement.

CHAPTER VII.

HINTS ON DESIGN AND ON TREATMENT OF THE INSTRUMENT.

It may perhaps be useful to offer a few hints upon the results which may be expected from various adjustments in combination.

All the designs which the Epicycloidal Cutting Frame can produce, and they are practically infinite, will depend upon the direction of motion (direct or retrograde), and upon concurrent values of

a, the excentricity of the Flange,
b, " " Frame,
x, y, } the two change wheels,

and of C, or **C**, the angular position, prescribed at the tangent wheel, of the curve whose size and form are defined by the preceding elements.

It will be convenient to consider x and y as unchanged for the time being, and C to have only such values as are necessary for "correction." This will reduce the number of varying quantities; leaving to be dealt with only a, b, and the direction of motion. And, in reviewing their practicable combinations, a better idea will be obtained by proceeding systemati-

cally, and giving, by way of experiment, to a and b respectively, certain assigned values in some regular order of variation.

It would seem that if we start from any definite values of a and b, the increment adopted being uniform, and those cases being omitted where a and b vary indiscriminately, the changes which are available are these :—

1. (a) is constant, while (b) diminishes . . A
2. ,, ,, increases
3. (b) ,, ,, (a) diminishes . . . B
4. ,, ,, increases
5. $(a + b)$ is constant, (a) diminishes, while (b) increases *pro tantô* C
6. $(a + b)$ is constant, (b) diminishes, while (a) increases *pro tantô* . . . . D
7. $(a - b)$ is constant, (a) and (b) each diminish equally . . . . E
8. $(a - b)$ is constant, (a) and (b) each increase equally.

If $(a + b)$ must not be greater than some fixed quantity (a stipulation which has to be made in practice very frequently), and if the maximum values of (a) and (b) be selected in the first instance, it follows that (a) and (b) cannot increase simultaneously, and then Nos. 2, 4, and 8 of the above are impossible. But the effects they would yield would not be greatly dissimilar from those of the companion changes Nos. 1, 3, and 7.

A further limitation would arise by providing that (b) shall never be greater than (a), and by continuing each prescribed variation only so far as may be advan-

tageous with reference to the design. While on the other hand the range of results would be extended by withdrawing any of the above restrictions; by selecting other values for (a) and (b) at the commencement; and by allowing their increase or diminution to be itself variable instead of uniform; to be irregular, or intermittent.

As an assistance to similar investigations, we will trace the results of the several changes marked A to E, in the statement just given, for the wheels $x = 48$, $y = 30$, (24 loops, V = 4·8), taking both directions of motion into account, i.e. having loops both external and internal. It is reasonable to suppose that the development will be more interesting when the points of departure for (a) and (b) are such values as refer to some special feature in the curve; its cusped, or polygonal, form for instance, or when it passes through the centre. In the present case, (a) and (b) shall receive the values which belong to loops in contact, and shall be as large as possible, without introducing fractions, so that $(a \quad b)$ does not however exceed 120 divisions: these values were found experimentally to be $a = 90$, $b = 30$ for internal loops; and $a = 80$, $b = 39$ for external loops.

Both kinds of compensation will be required: and for this we have

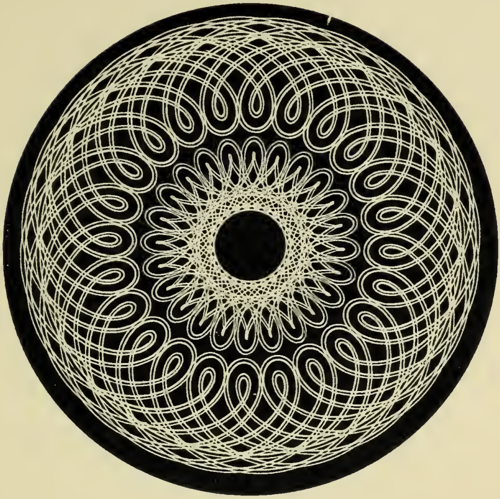
$$\mathbf{c} = \frac{2}{3} \times \frac{y}{x} = \frac{2}{3} \cdot \frac{30}{48} = \frac{5}{12} = 0{\cdot}42,$$

and

$$\mathrm{C} = 4 - \mathbf{c} = 3\frac{7}{12} = 3{\cdot}58.$$

The diagrams which follow are distinguished by the letters of reference affixed to the conditions stated above.

Fig. 78.



A. —— (internal). (*a*) constant. Fig. 78.

a	*b*	C	
90	30	none	border.
,,	28	,,	
. .	. .	. .	
. .	. .	. .	
,,	22	,,	
,,	20	,,	

A. —— (external). (*a*) constant.

a	*b*	C	
38	19	none	centre.
,,	17	,,	
. .	. .	. .	
. .	. .	. .	
,,	11	,,	
,,	9	,,	

This is not a bad example of the facility with which an elegant and apparently intricate design can be produced by the Epicycloidal Cutting Frame from very simple adjustments. Although no compensation is required, precaution is necessary, by the help of the tangent wheel, to obtain the symmetrical position of the opposing loops with reference to one another (see fig. 66, *suprà*). As regards the last two values for b, it may be observed that $V = \frac{24}{5} \therefore n = \frac{19}{5}$; and for cusps $\frac{a}{b} = n = \frac{38}{10}$, giving a value for b, when $a = 38$, intermediate to the two selected.

B. —— (internal). (b) constant. Fig. 79.

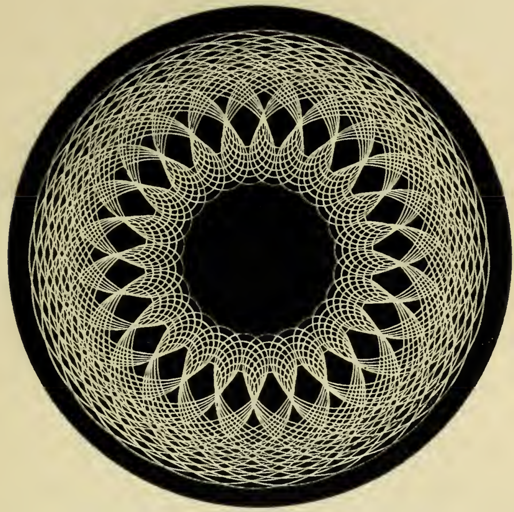
a	b	C t	C d
90	30	6	22·3
86	,,	,,	8
82	,,	5	43·6
78	,,	,,	29·3
74	,,	,,	15
70	,,	,,	0·6

$$C = 3\tfrac{7}{12} \therefore 4\,C = 14\tfrac{1}{3} = 14{\cdot}33$$
$$\text{and} \quad 70\,C = 250{\cdot}6 = 5 \quad 0{\cdot}6$$

The first line contains the values for a and b which have been assumed as the foundation for the experiment in progress; but the adjustments were made in the inverse order, beginning with $a = 70$, so that the tangent screw might be moved in the direction in which its graduations increase.

The values in column C are those which would maintain the position of the curves with respect to a

Fig. 79.



vertical line. But where the loops are, comparatively, so numerous, and intersect so frequently, as here, the definite position of the first curve may be disregarded; and the compensation required is simply $14\frac{1}{3}$ divisions of the tangent screw between every two of those succeeding.

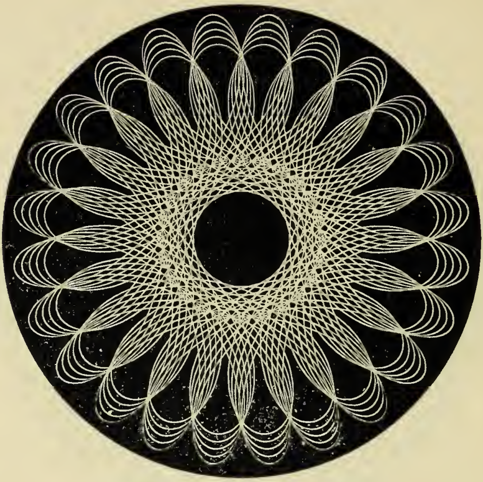
B. —— (external). (*b*) constant. Fig. 80.

a	*b*	**c**
80	39	$33\frac{1}{3}$
76	,,	$31\frac{2}{3}$
72	,,	30
68	,,	$28\frac{1}{3}$
64	,,	$26\frac{2}{3}$

$$\mathbf{c} = \tfrac{5}{12} \therefore 4\,\mathbf{c} = 1\tfrac{2}{3}$$

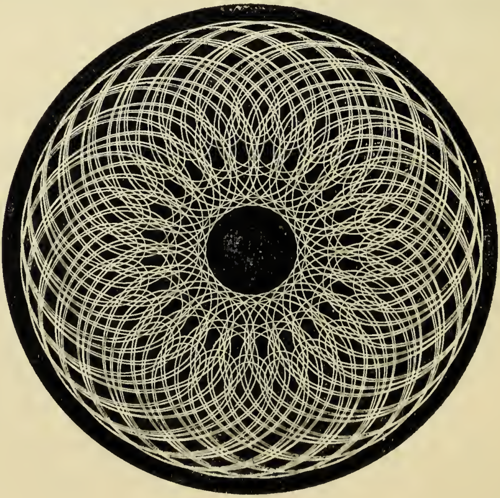
$$\text{and} \quad 64\,\mathbf{c} = 26\tfrac{2}{3}$$

Fig. 80.



The loops are here too prominent for the "initial position" to be neglected: the compensation for the 64 divisions through which the Flange was first moved is therefore included.

Fig. 81.



C. —— (internal). ($a + b$) constant. Fig. 81.

a	b	C t	C d
90.	30.	1.	17·3
88.	32.	,,	10·14
..	..	..	
..	..	..	
82	38	0.	38·7
80	40	,,	31·5
..	..	..	
..	..	..	
74	46	0.	7·16
72	48		

$$C = 3{\cdot}58 \therefore 2\,C = 7{\cdot}16$$

No compensation is necessary for the first curve. The progression is here intermittent, and produces, with slight trouble, an agreeable Tartan effect, of which fig. 73 was also an illustration, and which can only be attained by great care in ordinary Excentric Turning.

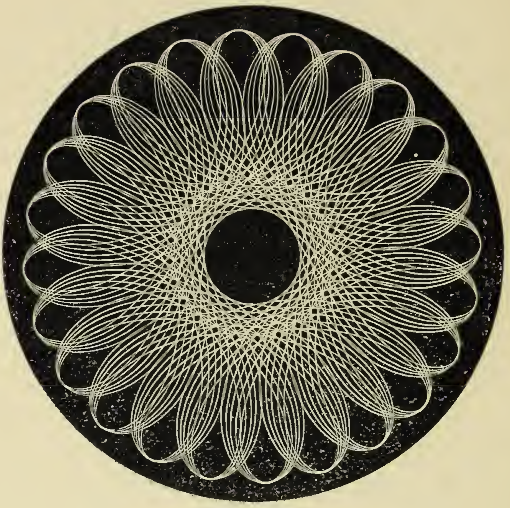
C. —— (external). ($a + b$) constant. Fig. 82.

a	b	**c** d
80	39	33·3
78	41	32·5
76	43	31·7
74	45	30·8
72	47	30

$$\mathbf{c} = {\cdot}42 \therefore 2\,\mathbf{c} = {\cdot}84$$

$$\text{and } 72\,\mathbf{c} = 30.$$

Fig. 82.

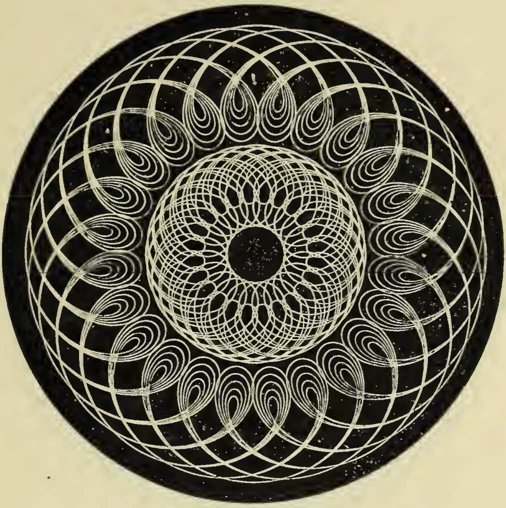


The resemblance between the centres of figs. 80 and 82, where the intersections are in each so close and numerous, is rather singular.

D. —— (internal). (a + b) constant. Fig. 83.

a	b	C t	C d	
90	30	6	22·2	border
92	28	,,	29·4	border
94	26	,,	36·6	border
96	24	,,	43·8	border
98	22	,,	7·1	border
36	21	2.	29	centre
42	15	3.	0·5	centre
48	9	,,	22	centre

Fig. 83.



$$2\,C = 7{\cdot}16 \text{ as before}$$
$$36\,C = 129 = 2^t,\ 29^d.$$

D. —— (external). $(a + b)$ constant. Fig. 84.

a	b	**C** t	d	
80	39	0.	$33\frac{1}{4}$	border
83	36	,,	$34\frac{1}{2}$	
86	33	,,	$35\frac{3}{4}$	
89	30	,,	37	
92	27	,,	$38\frac{1}{4}$	
22	16	0.	$9\frac{1}{4}$	centre.
28	10	,,	$11\frac{3}{4}$	

$$\mathbf{C} = \tfrac{5}{12} \text{ as before}$$
$$\therefore 3\,\mathbf{C} = 1\tfrac{1}{4}$$
$$\text{and } 22\,\mathbf{C} = 9{\cdot}16.$$

Fig. 84.

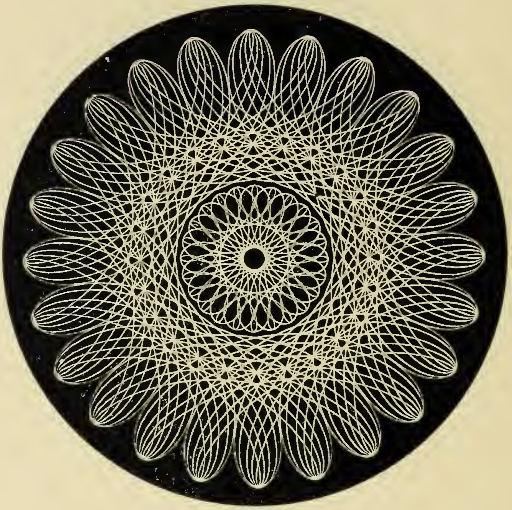


Fig 85.

E. —— (internal). $(a - b)$ constant. Fig. 85.

a	b	C t	C d	
90	30	6	$22\frac{1}{2}$	border
87	27	6	$11\frac{3}{4}$	border
84	24	6	1	border
81	21	5	$40\frac{1}{4}$	border
78	18	5	$29\frac{1}{2}$	border
36	21	2	29	centre.
30	15	2	$7\frac{1}{2}$	centre.
24	9	1	36	centre.

$$C = 3\tfrac{7}{12} \therefore 3\ C = 10\tfrac{3}{4}.$$

E. —— (external). $(a - b)$ constant. Fig. 86.

a	b	**C** t	**C** d	
80	39	0	$33\frac{1}{2}$	border
77	36	,,	$32\frac{1}{4}$	border
74	33	,,	31	border
71	30	,,	$29\frac{3}{4}$	border
68	27	,,	$28\frac{1}{2}$	border
65	24	,,	$27\frac{1}{4}$	border
62	21	0	26	border
22	16	0	$9\frac{1}{4}$	centre.
16	10	,,	$6\frac{3}{4}$	centre.

3 **C** $= 1\frac{1}{4}$, as in fig. 84.

When $(a + b)$ is constant, all the curves must clearly touch the "apocentral" circle; and, similarly, when $(a - b)$ is constant, they must all touch the "pericentral" circle. The filling in of the centres of this and the preceding specimens of the set has been restricted

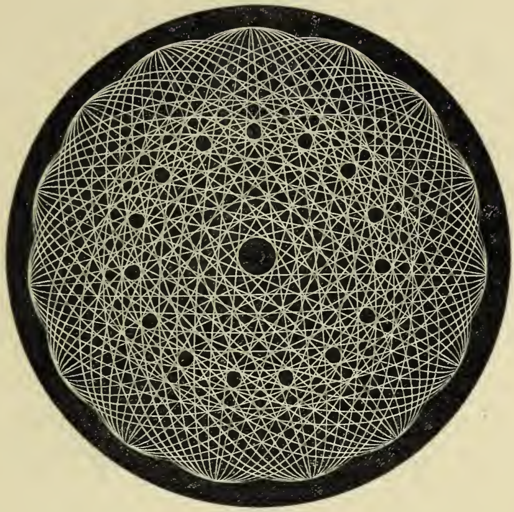
Fig. 86.

to figures of similar construction to the border, obtainable by the change-wheels which have been used throughout. For ornamental purposes others, perhaps of the same numbers of loops, but of a different class, might be substituted with advantage.

The next design (fig. 87), though very different in appearance, was produced by the same change-wheels 48 and 30; their order, however, was inverted, x being now $= 30$, and $y = 48$, reducing V from 4·8 to 1·87, and the loops from 24 to 15. The features of ellipse and straight line in circulation are plainly visible, as would be anticipated from the fact that V is here nearly equal to 2, the motion being negative.

In the figure, $(a + b)$ was constant, and the point of departure was $a = b = 60$. Those terms, however, were omitted, and the first curve was traced with $a = 55$, $b = 65$, the series being continued to $a = 20$, $b = 100$. The compensation was applied between the

Fig. 87.



adjacent curves, and was obtained, as usual, from the formula

$$\mathbf{c} = \frac{2}{3} \cdot \frac{y}{x} = \frac{2}{3} \cdot \frac{48}{30} = \frac{16}{15};$$

that is to say 16 divisions of the tangent-screw were required to correct 15 at the Flange, or $5\frac{1}{3}$ at the Tangent for 5 at the Flange, which were the adopted intervals. This is also a design which might probably be "tartanised" with advantage. Additional curves between the first and second, the second and third, the fourth and fifth, and the seventh and eighth, would give a richer character to the decoration, and would bring out more prominently the interesting circular group of fifteen untouched spots.

Instead of maintaining, as in the recent examples, a parallel arrangement of similar curves, any special

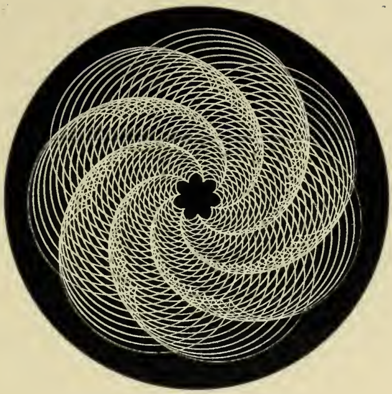
ratio for $\frac{a}{b}$ may be continued by using any convenient multiples of the numbers first assigned. In this manner loops of definite proportions may be repeated in various sizes. A familiar example is afforded by the ellipse, which, when a and b are diminished equally, assumes a less and less "excentricity," till at length it becomes a circle; and which, by a gradation of parallel ellipses of greater and greater "excentricity," assumes the form of a straight line, when the greater of the two (a or b) is diminished, the other (b or a) remaining constant; while, if the alteration in a and b be made in the same proportion as that which subsisted between them in the first instance, the ellipse can be made larger or smaller, *ad libitum*, still preserving its shape, though changing its size.

For example, the curves may be successively re-

Fig. 88.

peated, in the form where the loops have vanished into cusps, $\left(\frac{a}{b} = \pm\ n\right)$, and their uniformity of position may

Fig. 89.



be secured by the formulæ for compensation, as in fig. 88, or they may be distributed spirally, as in fig. 89.

Fig. 88. $x = 40$, $y = 36$; one carrier; loops (10) external:

$$V = 3\tfrac{1}{3},\ n = \frac{7}{3} = \frac{a}{b}.$$

c $= \frac{2}{3} \times \frac{36}{40} = \frac{12}{20} = 0{\cdot}6 = 4{\cdot}2$ at Tangent for 7 on Flange.

$$\left(\frac{a}{b}\text{ constant}\right).$$

a = 14, 21, 28, &c. to 63
b = 6, 9, 12, &c., to 27.

Fig. 89. $x = 42$, $y = 36$; two carriers; loops (7) internal:

$$V = 3\tfrac{1}{2};\ n = \frac{9}{2} = \frac{a}{b}.$$

no angular correction; tangent-screw moved one turn (in opposition) between the adjacent curves.

$a = 11\frac{1}{4}, 13\frac{1}{2}, 15\frac{3}{4}, 18,$ &c., to 72
$b = 2\frac{1}{2}, 3, 3\frac{1}{2}, 4,$ &c., to 16.

$\left(\frac{a}{b}\right.$ constant).

The continuance of the ratio for $\frac{a}{b}$, which may be found to indicate Tangency of loops, as in fig. 90, does not seem to afford results so advantageous as those where, beginning from the same ratio, or leading up to it, $(a - b)$ is constant, as in figs. 86 and 91.

Fig. 90.

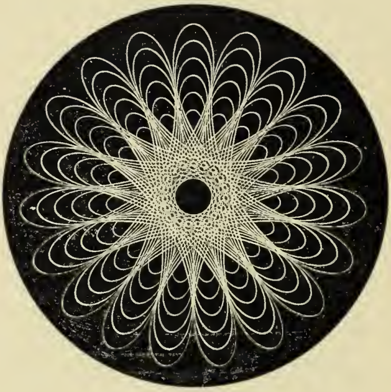


Fig. 90. $x = 38$, $y = 30$; one carrier; loops (19) external; for tangency $\frac{a}{b} = \frac{11}{7}$ by trial.

c $= \frac{2}{3} \times \frac{30}{38} = \frac{10}{19} = 0{\cdot}526 = 2{\cdot}89$ (say 3) at Tangent for $5\frac{1}{2}$ on Flange.

$\left(\frac{a}{b}\right.$ constant)

$a = 27\frac{1}{2}, 33,$ &c., to 55
$b = 17\frac{1}{2}, 21,$ &c., to 35.

Fig. 91.

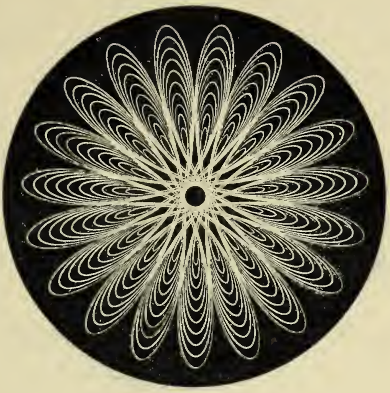


Fig. 91. $x = 38$, $y = 42$; one carrier; loops (19) external; $V = 2{\cdot}71$: the loops being longer, and approaching the centre more nearly, than in the last example, where, though the loops are also (19), $V = 3{\cdot}8$.

$\mathbf{c} = \frac{2}{3} \times \frac{42}{38} = \frac{14}{19} = 0{\cdot}74 = 2{\cdot}2$ at the Tangent for 3 on the Flange.

$(a - b)$ constant.
$a = 21, 24, 27$, &c., to 48
$b = 15, 18, 21$, &c., to 42.

The centre of this figure is rendered rather less tame than it would otherwise appear, by a slight variation in the second curve from the values just stated; b having been there taken at 17 instead of 18.

But these experiments might be continued indefinitely: they have been carried far enough to show that an inexhaustible store of effective combinations awaits the ingenuity of the amateur.

As regards the Instrument itself, and its manipulation, one or two points deserve attention.

The error known as "loss of time" is of no consequence in the screw by which excentricity is imparted to the Flange, since the graduations indicating that excentricity are read by marks on adjacent surfaces. It would have been of much advantage if a micrometer head could have been applied to the screw, or a vernier to the limb; but, as the divisions are unequal, these contrivances are inadmissible. To the Screw of the Eccentric Frame, however, this error is important, and should be counteracted, where it exists, by propelling the tool box in that direction in which the graduations are being reckoned for the occasion. (See Note 3, page 9, of Captain Ash's Treatise on "Double Counting" referred to in the Preface.) The point of the tool should also be central when the screw of the Eccentric Frame is at zero; and if, owing to unequal grinding of the bevelled edges of the tool, or to wear in the screw, this coincidence should not be accurate, the error must be ascertained and allowed for.

The milled-edged nut, which secures the two change-wheels upon their arbor, must be tightened (with the fingers) so firmly as to prevent its shaking loose from the motion of the instrument. And these wheels, as well as the carriers, should be set as deeply "into gear" as is consistent with smoothness of action, and the avoidance of excessive friction. If this precaution be neglected, there will be inevitably some excess of "play" among the wheels, causing "loss of time" throughout the train. This will probably be of little moment so long as the excentricities are of moderate amount, but will cause much unsteadiness when the combined motion is characterised by abrupt and considerable changes. Fig. 64—the star with forty-five radiations—illustrates this defect very plainly, and for

this reason has been allowed to remain in its imperfect form.

The teeth of all the wheels, and the principal bearing or collar of the instrument immediately behind the pulley, should be plentifully supplied with good oil.

The tangent-screw and wheel demand every care, because upon their efficiency depends the successful regulation of the entire mechanism; for a movement of less than half a division of the micrometer head of this screw, that is to say, about the one ten-thousandth part of the circumference of the tangent-wheel—makes, with some change-wheels, a very perceptible difference in the angular position of the curve. "Loss of time" is here of more serious consequence, and is guarded against by the provision of two pairs of adjusting-screws, which maintain a proper degree of pressure, by the tangent-screw, upon the edge of the wheel. This screw should be sufficiently tightened in its end bearings; and its threads, as well as the recessed teeth of the wheel, should be slightly lubricated with tallow. At the same time, it is not safe to depend on the supposed elimination of this error, and to move the screw both backwards and forwards, still assuming its readings to be uniformly correct. On the contrary, the screw should be kept moving in one direction so long as the zero of the "initial position" of the instrument remains unchanged; and that "initial position" should be arrived at by moving the tangent-screw in the direction in which it is about to turn during the execution of the design.

When large excentricities are employed, it is sometimes annoying to find that, on approaching the tool to the work, it leaves one portion untouched. It is manifestly essential that the point of the tool should move

in a plane parallel to the surface on which it is to operate; and the discrepancy just stated can only arise from a failure in that condition. If the upper and lower slides of the slide-rest are at right angles to one another, and the lower slide is also at right angles to the axis of the mandrel—adjustments as to which there need be no uncertainty—the tool cannot fail to reach both *sides* of the work; and it will be either the upper or the lower part of the latter which escapes the tool—that is, the pulley is inclined, instead of being vertical. This error in position may be caused by the intrusion of dust, or other small particles, between one edge of the sole of the rest and the bearers of the lathe, or between the square stem of the instrument and the surface of the receptacle upon which it rests; or there may be extreme pressure by one of the clamping-screws of the latter, or an over-tension of the band which connects the pulley with the overhead motion-shaft. Examination in these respects will generally suffice; but, if not, the necessary parallelism may be readily obtained by placing a small strip of the thinnest tinfoil, or even of tissue-paper, underneath the square stem in the receptacle, at whichever end such packing may be requisite.

When the excentricity of the Frame is very great, and that of the Flange very small, as under the circumstances detailed in Chapters IX. and X., another absence of parallelism may be rendered manifest by the loops, which are successively traced, being all touched more deeply on one side than the other. As this occurs irrespectively of the position which the loops occupy upon the surface where they are described, it points to a defect in the instrument itself—viz., that the Frame and the Flange are not moving in parallel planes. An

alteration in the pressure of the binding-screw of the Flange, or of the screw which tightens the latter upon its radial arbor, will probably correct this discrepancy; but the error may arise from a slight accumulation of dust between the back of the curved edge of the Flange and the surface against which it moves, or even from unequal wear of those surfaces. In the latter case some very slight packing may serve as a temporary expedient; but perhaps, in addition to the alternative holes, into either of which the milled-edged binding-screw can be fixed at pleasure, a third position might be advantageously provided, near the commencement of the scale, for use when the excentricity of the Flange does not exceed fifteen divisions.

A caution not to let the instrument fall is hardly so absurd as it may seem. The weight of the mechanism in front makes the whole apparatus top-heavy, and should it be released in the receptacle of the slide-rest without the hand being ready to give support, the instrument may drop forwards unexpectedly, and receive serious damage from collision with the bearers of the lathe.

The tool is continually changing the angle at which its cutting-edge is impelled, and, under some conditions, can only cut backwards, if at all, at certain points of its course. A very thin edge, as well as a very sharp one, will help to overcome this inconvenience.

The instrument will require cleaning occasionally, and to be taken to pieces for that purpose. In replacing its component parts, much care is necessary when handling the screw-driver and the lever for the capstan-headed screw behind the tangent-wheel, in order to obtain close fitting without too much pressure. For this, when excessive or unequal, may induce tor-

sion, interfering with freedom of action and the satisfactory performance of the instrument. The reputation of its makers is an ample guarantee that the permanent adjustments—such as the parallelism of Flange and pulley, the centrality of the Frame axis, the three sets of graduations, and the accuracy of wheel-cutting—were all perfectly attained during its manufacture; and reasonable care will maintain these necessary conditions unimpaired.

The high speed at which the Drill and the Eccentric Cutter are usually driven is not essential for the Epicycloidal Cutting Frame, and would only tend to needless wear and tear of the instrument. The groove of least diameter in the large "double-bevil" driving wheel of the lathe, and the largest in the pulley of the overhead shaft, will generally be the most appropriate position for the driving-band; but as the revolutions of the tool compared with the pulley will vary considerably in speed with the change-wheels employed, the same driving velocity will not be always equally suitable.

In preparatory trials both time and material may be spared by first tracing the intended pattern with pencil and paper. To receive the latter, a piece of inch "pine" board, about 9 inches by 6, with the corners rounded to adapt it to the height of the lathe centre, may be attached by wood screws to a brass flange chuck, and then be screwed upon the mandrel, direct, or with the intervention of the Eccentric Chuck. The surface of the wood should be corrected with the slide-rest, and a sheet of paper can be fastened to it by ordinary "drawing pins." A lead pencil in thin wood can be used, but it is difficult to fix, and almost impossible to centre. A small spring holder may be made to fit the tool-box of the Eccentric Frame, and to

receive a short length of one of the thicker sizes of "prepared leads," or, preferably, a small pointed cylinder of the metallic composition used for writing in indelible memorandum books, which can be procured in slips, with paper to correspond, from the wholesale stationers.

But this expedient, however useful for obtaining ideas of general effect, is thoroughly unsuited to accurate investigation; and, if adopted with that view—as, for example, to verify the compensation formulæ—would give uncertain and erroneous results

CHAPTER VIII.

EXTENSION OF THE INSTRUMENT, BY THE INTRODUCTION OF A SECOND PAIR OF CHANGE WHEELS.

COMPLETE as are the resources of the Epicycloidal Cutting Frame in its present form, they may easily be carried further; and, up to a certain point, with advantage. The most obvious addition is to increase the number of change wheels, from which two are to be selected, within the dimensions for which space can be found. The five intermediate sizes in even numbers, continuing the series from 48 to 60, would first be added to the set; and would afford some excellent combinations within the range of values for V lying between 5 and 6, of which Table III. offers but few examples. Thus, a wheel of 52 teeth would give $\frac{x}{y} = \frac{52}{30}$, or a curve of 26 loops, where V = 5·2. The fifteen odd numbers from 31 to 59, or some of them, might also be useful for some occasions: for instance, if we are able to put $\frac{x}{y} = \frac{35}{60}$, we have $V = \frac{7}{4}$, and the 7-looped figure thus produced differs considerably in some of its phases, from the only one of the same number of loops given in the preceding Tables.

But so long as we are restricted to the extremes of 30 and 60, which cannot be judiciously exceeded while the effect of the train is one of acceleration, "consecutive" loops of higher numbers than 6 are unattainable,

and V, in figures of circulation, cannot reach that value.

After some practice with the instrument, the amateur will probably be desirous of passing this limit of 6; and, to do this, it will be necessary to provide another pair of change wheels; which may be interposed, when required, between the pair denominated x, y, and the 60, or the 40, with both of which they have been hitherto connected. These extra wheels, which we will at once designate x', y', would be carried by a second removeable arbor, supported in any convenient manner which will afford the requisite facilities for adjustment of distance according to the diameters of the wheels employed. Such an additional arbor may be mounted upon a second steel plate, moving concentrically with the first (marked E in the frontispiece engraving), and clamped thereto; or, as was preferably adopted by Messrs. Holtzapffel when carrying out for the author this suggestion, the second plate may move radially upon the external socket of the Flange. In either case, the two arbors can be placed suitably with regard to all the wheels of the train; the radial plates can be secured in appropriate positions by their respective binding screws; and the continuity of the gearing will remain unaffected by any change in the excentricity of the Flange. The attached $\frac{32}{60}$ wheels, which are driven by the "carriers" (one or both), will need approximation, so as to make room laterally for the pair on the new arbor; and to accomplish this, without destroying the arrangement suited to the more simple form, a moveable "blank," or collar, with which the 60 wheel may be interchanged, is placed upon the hollow axis of the 32. Lastly, the former of the two "carriers" will require such a diminution in its projecting axis as will

permit that wheel, when geared with the 32, to pass easily behind the 60. The brass circular nut, with milled edge, of the second arbor should be of as small dimensions as may be practicable; and the similar nut of the first arbor will also have to be brought into less compass, in order that the wheels $x = 30$, $y = 60$, may be available, whatever may be the wheels on the second.

These slight alterations in the mechanism as first described and figured, and the addition of *three wheels only*, viz. 54, 50, and 30, to the original set as enumerated on page 4, will effect a considerable increase in the capabilities of the Epicycloidal Cutting Frame. Curves of 7, 8, 9, and 10 "consecutive" loops, and the singular one-looped figure, perhaps more interesting than ornamental, known in its cusped form as the cardioid, together with the large assortment of "circulating" figures yielded by four change wheels, with an assignable value for V extending as far as 10,—are now within the province of the instrument.

Some of the more prominent results are briefly indicated in Table V., page 134. Here, and subsequently, the wheels distinguished by x, y, are intended for the first, or original, arbor, which we will call A; and x', y', for the second, or additional, arbor, which we will call B. The former, A, is the one which is now carried by the new radial steel plate, and which receives the pair of change wheels whereof one drives the 40 on the axis of the Eccentric Frame.

This new plate, however, does not require any radial movement at all, and is really better without such a facility, being screwed permanently to the Flange in such a position that the line of direction of its longitudinal mortise would pass through the centre of the 40 wheel on the Frame axis. One arbor, A, will pass along this

mortise until the teeth of the wheel x encounter at a proper depth those of the 40 wheel on the Frame axis: and the other arbor, B, will in like manner slide in the mortise of the original radial plate until the teeth of the wheel y' and of the 60 wheel on the radial arbor are suitably engaged. It then remains to move this radial plate forwards on its centre, until the wheel x' on B meets correctly the wheel y on A—the adjustment for "initial position" being borne in mind at this point—and the whole train of gearing is then complete. The three binding screws, which maintain the positions of the two removeable arbors, and of the original radial plate, should be well secured, so as to obviate the tendency of the wheels to become disengaged during their revolution.

As a guide for the convenient distribution of the change wheels, it may be remarked that, when their effect is to accelerate, the *least* of the four should be placed upon A, and the *greatest* of the four upon B; but, when their effect is to retard—a condition on which the next chapters depend—then the *least* should be placed upon B, and the greatest upon A. In some cases this order of arrangement is not material; but it is always to be understood that either y or y' is placed in contact with the 60 on the arbor which forms the Flange centre; and either y or y' with the 40 on the axis of the Eccentric Frame. So that the effect of the whole train, i.e. V, is now expressed by $\frac{3xx'}{yy'}$.

Besides those given in Table V., other effective arrangements will doubtless present themselves in course of practice or by previous calculation. The labour of tabulating beforehand all possible combinations, especially if further change wheels be provided,

TABLE V.

	Loops	V	R	Arbor A x	Arbor A y	Arbor B x'	Arbor B y'	C_2*	C_2
Consecutive	1	1	1	30	54	30	50	2	—
	7	7	1	42	30	50	30	0·28	4·28
	8	8	1	48	30	50	30	0·25	4·25
	9	9	1	50	30	54	30	0·22	4·22
	10	10	1	50	30	60	30	0·2	4·2
Circulating	15	7·5	2	50	30	60	40	0·2$\dot{6}$	4·2$\dot{6}$
	23	7·6$\dot{6}$	5	42	30	50	30	0·26	4·26
	25	8·3$\dot{3}$	3	50	30	60	36	0·24	4·24
	27	6·75	4	50	30	54	40	0·29	4·29
	42	8·4	5	42	30	60	30	0·24	4·24
	48	9·6	5	48	30	60	30	0·21	4·21

would be considerable, and would occupy a needless amount of space; but the following remarks may assist the amateur to determine what curves are within the compass of his instrument, and how to select wheels by which they can be produced.

It will be remembered that when the curve is of the class called "circulating," V, which stands for the value of the whole train of wheels, and denotes the ratio between the velocities of Flange and Frame, is represented by a fraction whose numerator expresses the number of loops in the curve. Therefore, having taken for the numerator the number of loops desired, we may adopt any figures we please for the denominator, provided the resulting fraction does not exceed 10, which it has been agreed shall be the limit of V: and the next step will be to ascertain whether the ratio thus prescribed for the two velocities can, with the wheels at our command, be imparted to the instrument.

* External loops are here produced with *both* "carriers," and internal loops with *one*. In the latter case, the compensation (C_2) is applied inversely,—that is, by turning the micrometer screw *backwards*, against its graduations. See page 140.

To discover what change wheels should be employed in any given case, we first multiply by 3 the denominator of the fraction representing V; because that is the accelerating effect of the permanent wheels of the train; and then, if possible, multiply both numerator and denominator by some convenient number, which will express the fraction in terms of two of the wheels at our disposal. For instance, if $V = \frac{7}{2}$, the change wheels must be represented by the fraction $\frac{7}{2 \times 3} = \frac{7}{6}$; or, multiplying both by 6, we have $\frac{x}{y} = \frac{42}{36}$, numbers with which we are already provided, and requiring one arbor only. But it will generally happen, when adopting the extended system, as for example when $V = 7$, and the change wheels must therefore be in the proportion of 7 to 3, that it is not practicable to obtain the required ratio by a single pair. In such cases, it is desirable to break up the fraction into two, separating the factors if any exist, and if not, multiplying its numerator and denominator by any number which seems promising; if necessary, repeating that process at discretion, till four numbers are obtained which correspond to four wheels in the set. Thus, in the present instance,

$$\frac{7}{3} = \frac{7}{3} \times \frac{5}{5} = \frac{7}{5} \times \frac{5}{3} = \frac{42}{30} \times \frac{50}{30},$$

of which fractions the two numerators are to be taken for x, x', and the two denominators for y, y'. Similarly, if $V = \frac{7}{3}$, we get $\frac{x}{y} = \frac{7}{9} = \frac{42}{54}$, a proportion for which the single arbor may be used, since 54 is one of the three new wheels recommended. But if $V = \frac{7}{4}$, $\frac{x}{y}$ becomes $= \frac{7}{12}$, and this cannot be expressed by one

pair (unless the 35 wheel happens to have been added, when we should have $\frac{35}{60}$); and it will therefore be necessary to have recourse to the method of subdivision. Here, either of the numbers 7 or 8 appears to be a convenient multiplier; and we may take

$$\frac{xx'}{yy'} = \frac{7}{12} = \frac{7 \times 8}{8 \times 12} = \frac{42}{48} \times \frac{40}{60};$$

or
$$\frac{xx'}{yy'} = \frac{7}{12} = \frac{7 \times 9}{9 \times 12} = \frac{42}{54} \times \frac{36}{48}.$$

When curves which may be produced by one arbor only are associated with others requiring four change wheels in the manner described, it will be desirable, in order to avoid a too frequent disturbance of the instrument, that two equal wheels be placed upon one of the arbors; or, if that be inconvenient, to calculate another arrangement of wheels requiring both arbors, by subdividing the fraction which expresses V, in accordance with the foregoing explanation. Another expedient, and the most satisfactory, is to interpose the "blank," with which the arbor B is provided, between its two change wheels, and to place upon the arbor A any convenient wheel (one of the two 48's for instance) which may serve as a "carrier" between x' and the 40 on the Frame axis, and may be secured on its arbor by adding, as a "blank," any small wheel which happens to be at liberty.

It may, perhaps, be thought that this method of extension has been abandoned too soon, and that facilities for a multiplying effect exceeding 10 might have been provided. There would be no difficulty in placing wheels in the proportion of 2 to 1 upon each of the moveable arbors, thus obtaining 12 "consecutive"

loops; and it would no doubt be possible to scheme even a third arbor if such a further addition were prudent. This, however, is not to be recommended. The simplicity of the Epicycloidal Cutting Frame, and the fewness of its adjustments, are not the least of its advantages; and if the author may venture so far, he would advise that the addition already suggested, or any other, be postponed until an intimate acquaintance with the instrument in its simple form has been acquired. When the office of a train of wheels is to retard, as in the Geometric Chuck, their number may be increased almost indefinitely, and there may be great difference in size between any driver and its follower, without seriously increasing friction, or interfering with steadiness of motion. But this is not so when the train accelerates the original velocity. Under those circumstances, the introduction of additional axes, and any great disparity between the drivers and the driven, would add largely to the friction—so much so, that it is possible for the power required to drive the train of wheels to exceed what will suffice to break them. Experiments in this direction will show that to give the Frame ten revolutions for one of the Flange is probably as strong a measure as can be adopted with safety, and that with a value for V at all approaching this ratio, much care is required in adjusting the depth of the teeth of the wheels, in the penetration of the cutting-tool, and in keeping the radius of the Eccentric Frame within moderate limits. A low driving velocity is also, in such a case, more essential than previously; and, to secure this, it may be worth while to support, in some temporary manner, from the lathe backboard, a pair of diminishing pulleys, to reduce, by one-half or more, the

speed which would otherwise be transmitted from the overhead motion shaft to the pulley of the instrument; or, as a simpler and better plan, to adapt to the crank-shaft of the lathe a pulley of much smaller diameter than the least of the grooves usually contained in the "double bevil wheel."

The obliquity in succeeding curves, due to a variation in the excentricity of the Flange, will, of course, continue when four change wheels are used; and it may be counteracted on the same principle, though not precisely in the same manner, as that which proved effectual when we were dealing with one arbor only.

The value of "the short train from S to M" (fig. 56) is now evidently expressed by $\frac{3xx'}{2yy'}$; but,* as there are now four axes in that train instead of three, the number of revolutions made by the Frame axis for each turn of the Flange in adjustment is consequently represented by $\left(1 + \frac{3xx'}{2yy'}\right)$ instead of by $\left(1 - \frac{3x}{2y}\right)$. In all other respects the reasoning of Chapter IV. holds good; the angle OSM = $0° \; 18'$, and the angular value of one division of the Tangent-screw micrometer remains the same, viz. $0° \; 4'{\cdot}5$.

But, since there is now another axis in the train, the office of the "carriers" becomes reversed: internal loops are now obtained with *one* carrier, and external loops with *both*. The two kinds of compensation are reversed as well, the greater of the two, though referring, as before, to the case where both carriers are employed, now belonging to the external loops.

* Willis's *Principles of Mechanism*, art. 404, page 325, Longmans, 1870; or Goodeve's *Elements of Mechanism*, page 160, Longmans, 1870.

And, for the same reason, the sign of the angle of displacement will be changed. When the epicycle is direct, the angle of correction will be equal to the difference (instead of to their sum, as when there were three axes from S to M) of the two angles P M C, C M T; and, when the epicycle is retrograde, the angle of correction will be equal to the sum (instead of to the difference) of the two angles P M C, C M *t*.

(i) Taking the latter case first, where there are now two carriers,—we have

$$\theta = n \cdot \frac{\text{O S M}}{2} + \left(\frac{3xx'}{2yy'} + 1\right) \text{O S M.} \quad . \quad . \quad . \quad (5)$$

The epicycle being retrograde,

$$n = \text{V} - 1 = \frac{3xx'}{yy'} - 1,$$

$$\therefore \theta = \frac{3xx' - yy' + 3xx' + 2yy'}{yy'} \cdot \frac{\text{O S M}}{2}$$

$$= \frac{6xx' + yy'}{yy'} \times 9'.$$

The same symbols, C and **C**, (with the distinction of the suffix 2, as now referring to *two* pairs of change-wheels,) may continue to denote the number of divisions to be moved at the Tangent-wheel for each division of the Flange, in order to correct the deviation arising from a change in the excentricity of the latter; C_2 being used with reference to internal loops, and $\mathbf{C}_2$ for external. Any angular interval through which the Tangent wheel may be moved, is, as before, multiplied by the effect of the whole train by the time it has reached the Frame axis, and we again have, to complete the present case :—

$$\mathbf{C}_2 = \frac{\theta}{\text{V} \times 4{\cdot}5} = \frac{6xx' + yy'}{yy'} \times \frac{9}{4{\cdot}5} \times \frac{yy}{3xx}$$

$$= \frac{2}{3} \cdot \frac{6xx' + yy'}{xx'} = 4 + \frac{2}{3} \cdot \frac{yy'}{xx'}$$

$$= 4 + \frac{2}{\text{V}} \cdot \quad \dots \dots \dots \dots \quad (6)$$

(ii) Then, for the direct epicycle, when there is now only one "carrier," and when

$$n = \text{V} + 1 = \frac{3xx}{yy} + 1,$$

we have

$$\theta = n \cdot \frac{\text{OSM}}{2} - \left(\frac{3xx'}{2yy} + 1\right) \text{OSM} \quad \dots \quad (7)$$

$$= \frac{3xx' + yy' - 3xx' - 2yy'}{yy'} \cdot \frac{\text{OSM}}{2}$$

$$= -1 \times 9',$$

and

$$\text{C}_2 = -\frac{9}{4{\cdot}5} \times \frac{yy'}{3xx'} = -\frac{2yy'}{3xx'}$$

$$= -\frac{2}{\text{V}} \cdot \quad \dots \dots \dots \dots \quad (8)$$

The minus sign here indicates that the correction is to be applied negatively at the Tangent wheel, by turning its micrometer screw *backwards*; a condition which might be inconvenient to reduce to practice were it not for the invariably small value of this correction. But if any difficulty should be experienced in this respect, it may be obviated by reading the *Flange* backwards, as being the easier operation of the two, the graduations of the micrometer screw then occurring in their natural order.

These formulæ are clearly analogous to those deduced for the single arbor (page 74): they are quite as simple as could be hoped for, where four change wheels are concerned, and will probably cause no difficulty in their application. Their equivalents are included in Table V. for the combinations of change wheels there given; and, for any others, if V be known beforehand, the compensation is known also; and if the change wheels be arbitrarily selected, the fraction $\frac{xx'}{yy'}$ in its lowest terms will be equally available.

Where one removeable arbor only was employed, we found an advantage in the circumstance that *the sum* of the two kinds of compensation, for all values of x and y, was invariably equal to 4. And, now that two such arbors are in use, we see that the same relation continues; but, in actual practice, regarding the negative sign as a precept for direction only, we may say that *the difference* of those quantities is constant, and equal to the same amount. Having therefore obtained the value of C_2 by the simple operation just described, we have only to increase that result by the figure 4, and it then becomes the value for $\mathbf{C_2}$.

Before practically applying the compensation, it is as requisite as formerly that the Flange be central and be at right angles with the Frame, the latter being perpendicular to the lathe bearers: and that the reading of the Tangent wheel be noted under these conditions. But it is sufficiently difficult when V = 6 to obtain an accurate adjustment of this "initial position:" and when, with a value for V lying between 6 and 10, the influence of the Tangent wheel at the Frame axis is so much greater than before, a satisfactory attainment of the zero point is still more hazardous, and the equable

transmission of the angular correction becomes more uncertain : another reason for not carrying too far this method of augmenting velocity.

Fig. 92 is an example of nine consecutive internal loops and cusps. The wheels $x = 50$, $y = 30$, $x' = 54$, $y' = 30$, give the required ratio between the velocities of Flange and Frame, and, in accordance with a previous recommendation for their arrangement, the former pair should be placed upon the arbor A next to the Frame axis, and the latter pair upon B. The wheels 50,

Fig. 92.

30, enter frequently into the composition of the trains required, and will on that account also be placed more suitably upon A, as that arbor need not be disturbed when changing the wheels upon B.

For the central cusps,

$$\frac{a}{b} = n = \frac{10}{1},$$

and the dimensions adopted were these :—

a =	5,	$7\frac{1}{2}$,	10,	$12\frac{1}{2}$,	$17\frac{1}{2}$,	20,	$22\frac{1}{2}$,	25,	$27\frac{1}{2}$,	30,	$32\frac{1}{2}$.
b =	$\frac{1}{2}$,	$\frac{3}{4}$,	1,	$1\frac{1}{4}$,	$1\frac{3}{4}$,	2,	$2\frac{1}{4}$,	$2\frac{1}{2}$,	$2\frac{3}{4}$,	3,	$3\frac{1}{4}$.
C_2 =	− 1·1	1·6	2·2	2·7,	3·8,	4·4,	4·9,	5·5,	6,	6·6,	7·1.

Only *one* of the "carriers" is now required, though the curve is internal; and it will be remembered that the compensation has to be reckoned *backwards*: the "initial position" must therefore be finally determined by turning the micrometer screw inversely. The amount of correction is seen at a glance to be

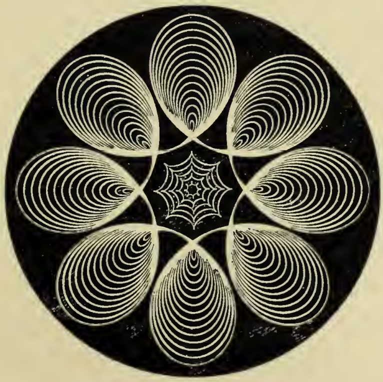
$$- \frac{2}{V} = - \frac{2}{9} = - \cdot 22$$

In the border, the reduction of the loops did not commence, until after a second curve had been described, parallel to that with loops in contact.

a =	65,	65,	$67\frac{1}{2}$,	70,	$72\frac{1}{2}$,	75,	$77\frac{1}{2}$,	80.
b =	28,	26,	$23\frac{1}{2}$,	21,	$18\frac{1}{2}$,	16,	$13\frac{1}{2}$,	11.
C_2 =	−	14·3,	14·8,	15·4,	16,	16·5,	17,	17·6.

The next example, Fig. 93, is of external form, for which both "carriers" must now be used.

Fig. 93.



There are sixteen curves of eight loops each, all situated on the same imaginary pericentral circle. The excentricities of the Frame and Flange were 6, and 30, divisions respectively, to begin with, and were increased by equal intervals of two divisions, till the values became $b = 36$, $a = 60$. Since V here = 8, the correction at the Tangent wheel was $4 + \frac{2}{8}$; i.e. $\mathbf{C}_2 = 4{\cdot}25$. This gave 2^t, $27\frac{1}{2}^d$ as the amount to be added to the zero point reading for the first curve, (a formality which can be omitted wherever the position of the pattern as a whole is not essential,) and for each of the subsequent intervals, $8\frac{1}{2}$ divisions of the micrometer screw provided the requisite compensation.

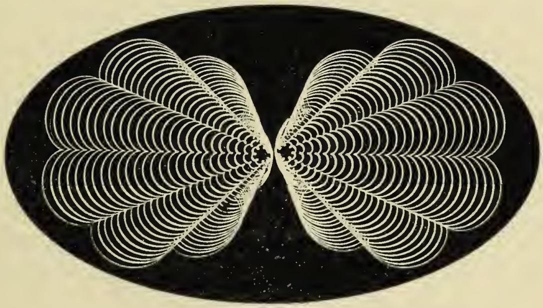
The central cusps were described without altering the arrangements already made, and were taken with reference to the same zero point. For the proportionate values of a and b, we have $n = \frac{7}{1} = \frac{a}{b}$: they commenced at $a = 3\frac{1}{2}$, $b = \frac{1}{2}$, and were continued to $a = 17\frac{1}{2}$, $b = 2\frac{1}{2}$, maintaining the same ratio. The Tangent wheel correction was again defined by $\mathbf{C}_2 = 4{\cdot}25$, and as the intervals at the Flange were now of $3\frac{1}{2}$ divisions, the corresponding adjustment at the screw of the Tangent wheel was $14{\cdot}9$ divisions.

Considerable caution is necessary with such high values for V as these, to guard against "loss of time" in the Tangent screw; and to this end, at least half a dozen turns should be taken, after its reversal for any purpose, before resuming its former direction. But even then, the recovery of the "initial position" is a matter of much uncertainty when V is comparatively so great: and it may be observed in the above figure that though the loops are symmetrically placed *inter se*, and the cusps are the same, yet the mutual position of

the two groups is not strictly accurate: the fact being that the cusps were traced after the loops were finished.

The double shell (fig. 94) is composed of a series of seven internal cusps, brought by the slide-rest and division-plate into the positions which they occupy. Since, here, $n = 8 = \frac{a}{b}$, the excentricity of the Flange must in all cases be eight times that of the Frame; and in order that each pair of curves may pass through the centre of the figure, the instrument will have to be moved sideways upon the slide-rest by such equal

Fig. 94.



intervals that the distance of its centre from the axis of the mandrel shall be always equal to $(a + b)$. A correction at the Tangent-wheel will be required for each pair of curves, to maintain their symmetrical position; and as $V = 7$, we have $C_2 = -\frac{2}{7} = -\cdot 285$. There are twenty-six lines in each shell; and the duplication was accomplished by moving the division-plate half-way round, and repeating the curve, before altering the successive adjustments.

The latter are four in number, and are given below for some of the first, and for the concluding curves; which will enable the amateur to complete, vary, or

extend the series at pleasure. The largest curves were described first; and then, as the reading at the Flange was reduced, that at the Tangent-wheel was increased.

a	= 54,	52,	50,	48,	. . . .	8,	6,	4
b	= $6\frac{3}{4}$,	$6\frac{1}{2}$,	$6\frac{1}{4}$,	6,	. . . .	1,	$\frac{3}{4}$,	$\frac{1}{2}$
C_2	= 0,	·57,	1·14,	1·7,	. . .	13·14,	13·7,	14·3
S·R	= 0,	$2\frac{1}{4}$,	$4\frac{1}{2}$,	$6\frac{3}{4}$,	. . . .	54,	56,	$58\frac{1}{2}$

The figures in the last line are divisions, and fractions of a division, of the micrometer head of the slide-rest screw, and are reckoned towards the centre of the work. They are expressed like those of the excentricities of Flange and Frame, in hundredths of an inch; and where parts of a division occur in so simple a form as here, they may be apprehended more readily as vulgar fractions than as decimals. This design, which would be more effective if the loops were more numerous, is a two-fold adaptation of one of the "compound patterns produced by rosette D," to be seen in an appendix to Messrs. Holtzapffel & Co.'s catalogue, as illustrating the capabilities of their Rose Cutting Frame. The effect produced by a succession of cusps, internal or external, obtained by an Epicycloidal Chuck or Cutting Frame, bears considerable resemblance to some examples of Rose Engine Turning;—with this difference, however, that by the former method the indentations which are produced are proportional in depth, and by the latter they are constant.

When the loops occur alternately, the effect is nearly as good as when they are consecutive. Fig. 95 gives an illustration of this; V being $\frac{15}{2}$. In the internal loops which form the border, the curves are parallel to

one another, and no compensation is needed. For the external loops in the centre we have

$$\mathbf{C}_2 = 4 + \frac{2}{V} = 4 + \frac{4}{15} = 4{\cdot}27.$$

Fig. 95.

a =	18,	20,	22,	24,	26,	external
b =	9,	11,	13,	15,	17,	loops in
$\mathbf{C}_2$ =	1 27·4,	1 36,	1 44·6,	2 3·2,	2 11·8,	the centre.

The five curves with internal loops were all described with a = 68, and the first was corrected

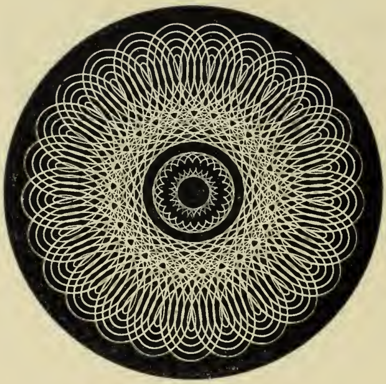
$$[C_2 = 68 \times (-\ {\cdot}27) = -\ 0\ 18{\cdot}4]$$

to obtain a correspondence in position with the inner group.

The general appearance of the next example (fig. 96) could be produced almost as well with one pair of change-wheels as with two. The number of loops, however, is here 25—one which Table III. does not afford. V is equal to $6\frac{1}{4} = \frac{25}{4}$, therefore $n = \frac{29}{4}$ for internal cusps, and $\frac{21}{4}$ for external. These being

the ratios which must subsist between a and b, when the curve assumes that condition, the values taken for the two single lines in the centre of the figure were $a = 15\frac{3}{4}$, $b = 3$ (external), and $a = 21\frac{3}{4}$, $b = 3$ (internal). The large external loops which form the border were brought, by trial, to alternate contact:—a was 58 for all the five curves, therefore no compensation was needed; and b was reduced, by four divisions at a time, from 31 to 15.

Fig. 96.



The change-wheels employed were $x = 50$, $y = 30$; $x' = 60$, $y' = 48$.

Another arrangement of the same number of loops can be obtained, where

$$V = 8\tfrac{1}{3},\ [x = 50,\ y = 30 : x' = 60,\ y' = 36] \text{ fig. 97.}$$

Since $V = \frac{25}{3}$,

$$C_2 = -\tfrac{6}{25} = -0{\cdot}24;\ \text{and}\ \mathbf{C}_2 = 4{\cdot}24.$$

$$\left.\begin{array}{lll} a = & 24, & 22, \\ b = & 4, & 6, \\ C_2 = & -5{\cdot}8, & -5{\cdot}2, \end{array}\right\} \text{centre of the figure.}$$

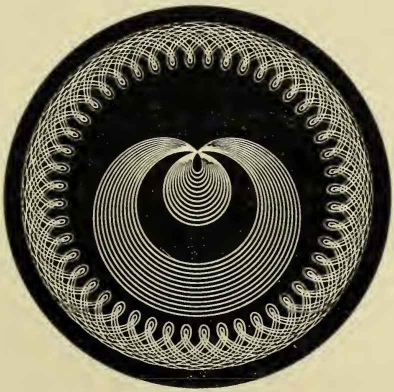
Fig. 97.

a =	40,	46,	52,	58,	64,	70,	
b =	10,	12,	14,	16,	18,	20,	external loops.
C_2 =	3 20,	3 45·4,	4 20·8,	4 46·2,	5 21·6,	5 47,	

One of the highest values which V can receive from the additional pair of change-wheels is 9·6; the loops being 48 in number.

$$[x' = 60, y' = 30, x = 48, y = 30.]$$

Fig. 98.



The border of fig. 98 consists of three curves of this description,

$$a = 80, \; b = 7\tfrac{1}{2}, \; 9\tfrac{1}{2}, \; 11\tfrac{1}{2};$$

the excentricity of the Flange was unaltered, and no compensation was required.

The figure which the border encloses is a succession of the one-looped curves produced when $V = 1$, and the Flange and Frame are moving in the same direction. Consequently, one of the "carriers" was dispensed with for the whole of this design. A correction at the Tangent-wheel is here very necessary as the series of loops proceeds; for

$$C_2 = -\frac{2}{V} = -2.$$

The excentricity of the Flange was diminished by intervals of $2\frac{1}{2}$ divisions from 40 to 10; therefore the reading of the micrometer screw was increased for each curve in succession by twice that amount, or 5 divisions: the excentricity of the Frame was equal to 25 throughout, and all the curves intersect at the same point. The figure assumes spontaneously the excentric position which it occupies, the instrument having remained, during the execution of the whole, in the line of centres to which it was first adjusted.

When the desired ratio between the velocities of Flange and Frame can be obtained by one pair of change-wheels, they may be placed on the arbor B, using a single wheel on A, in preference to removing B altogether and changing the relative positions of the "blank" and the 60 on the hollow axis of the 32. But it is important to observe that the tabular corrections, obtained from considerations depending upon a single

pair of change-wheels, cannot be used with the additional axis in the train from S to M: although, as now supposed, that axis receives a "carrier" only.* The formulæ for compensation under such circumstances will be those recently deduced, viz.:—

$$C_2 = -\frac{2yy'}{3xx'} \text{ where } \frac{y}{x} = 1.$$

That is, whether the arbor A carries two wheels or only one, we shall have

$$C_2 = -\frac{2}{V} \text{ and } \mathbf{c}_2 = 4 + \frac{2}{V};$$

instead of $\mathbf{c} = \frac{2y}{3x}$, and $C = 4 - \mathbf{c}$, as formerly.

To impress the more forcibly the fact that the compensation prescribed for one removable arbor will not serve for two, the following simple figure (Fig. 99) is here introduced, with adjustments, in complete detail, adapted to the case where both arbors are retained, the change-wheels being on one only.

$x' = 60$, $y' = 30$, $x = y$, *two* "carriers," $V = 6$, loops (6) external. Fig. 99.

$n = 5 = \frac{a}{b}$ for the series of cusps: "initial position" found, and "zero point" noted.

$$\frac{2}{V} = \tfrac{1}{3} \therefore \mathbf{c}_2 = 4 + \frac{2}{V}, = 4\tfrac{1}{3}, = \tfrac{13}{3};$$

or 13 divisions at the Tangent-wheel will compensate for 3 on the Flange. Therefore, as fractions are more easily estimated at the micrometer head of the Frame

* See page 136.

than at that of the Tangent-screw, and accuracy is more important at the latter, the values taken for *a* are multiples of 3, which will require multiples of 13 at the Tangent-wheel.

a	= 6,	9,	12,	—	18,	21,	24,	27
b	= 1·2,	1·8,	2·4,	—	3·6,	4·2,	4·8,	5·4
C_2	= 26,	39,	1·2,	—	1 15,	1 28,	1 41,	2 4

Fig. 99.

The Flange being again made central, one "carrier" was withdrawn, the "initial position" recovered, and the new "zero point" ascertained, care being taken to accomplish this adjustment by moving the Tangent-screw inversely. The compensation is now $-\frac{2}{V} = -\frac{1}{3}$, or one division must be moved backwards at the Tangent-wheel for 3 at the Flange; and *a* was therefore again taken by multiples of 3. The dimensions were :—

a =	39,	42,	45,	48,	51
b =	15,	15,	15,	15,	15
C_2 =	− 13,	14,	15,	16,	17

A similar course may be advantageously adopted in other cases where one pair of change-wheels would

suffice, in instruments which have been provided with an additional arbor, as suggested in this chapter. Successful performance depends in great measure upon the accurate centering of the Flange, and the latter should therefore be disturbed as little as possible. Perhaps when the instrument is specially arranged for a course of elliptical cutting, it may be restored to its simple form, and the second arbor may be dispensed with ; but for all other values of V both arbors may be employed with perfect convenience.

CHAPTER IX.

METHOD OF OBTAINING, BY MEANS OF THIS EXTENSION, CONSECUTIVE EXTERNAL LOOPS OF HIGH NUMBERS.

THERE are many different ways in which a curve of a given number of loops may be described; in fact, if that number be prime, and be denoted by N, there are (N − 1) different ways of producing such a curve. But it does not follow that they will be all within the compass of the instrument, nor that they will all possess a distinctive character. A curve of 7 loops, for example, may be of any of the six forms obtained when V is equal to any one of the fractions $\frac{7}{1}$, $\frac{7}{2}$, $\frac{7}{3}$, $\frac{7}{4}$, $\frac{7}{5}$, $\frac{7}{6}$: these six varieties, however, as will be seen presently, are, under certain conditions, reducible to three.

A curve of 24 loops, which appears to be frequently introduced in designs of simple geometric turning, cannot be obtained in a "circulating" form with V of higher value than $\frac{24}{5}$, (= 4·8, an arrangement included in Table III.), because 24 is divisible exactly by 2, 3, and 4; and the curve would, in those cases, be simply reduced to forms having 12, 8, and 6 loops respectively: of which the 8 and 6 only are within the reach of that extension of the instrument which has been suggested. If the desired number of loops be 25, the highest ratio we can take for V is $\frac{25}{3} = 8\frac{1}{3}$; since $\frac{25}{2}$, $= 12\frac{1}{2}$, exceeds the prescribed limit. Its equivalent

will be $\frac{xx'}{yy'} = \frac{25}{3 \times 3} = \frac{5 \times 5}{3 \times 3} = \frac{50}{30} \times \frac{60}{36}$ at once, without any arbitrary multiplication. (See fig. 97.)

Other varieties of the curve with 25 loops would be expressed by the quantities $\frac{25}{4} = 6\frac{1}{4}$, $\frac{25}{6} = 4\frac{1}{6}$, $\frac{25}{7} = 3\frac{4}{7}$, &c., which give a successively increasing width for the space occupied by each loop, while *a* and *b* remain constant and *a* is greater than *b*. When this ratio becomes nearly equal to 2, as

$$\text{to } \frac{25}{12}, \left[\frac{xx'}{yy'} = \frac{5 \times 5}{9 \times 4} = \frac{30}{54} \times \frac{60}{48}\right], \text{ fig. 100,}$$

the curve, as in many previous instances, when the loops are external, partakes of the elliptic character by repetition. In advancing beyond this point,

$$\text{to } \frac{25}{21}, \left[\frac{xx'}{yy'} = \frac{5 \times 5}{7 \times 9} = \frac{30}{42} \times \frac{30}{54}\right], \text{ fig. 101,}$$

$$\text{to } \frac{25}{16} \left[\frac{xx'}{yy'} = \frac{5 \times 5}{6 \times 8} = \frac{30}{36} \times \frac{30}{48}\right], \text{ fig. 102,}$$

$$\text{and to } \frac{25}{24}, \left[\frac{xx'}{yy'} = \frac{5 \times 5}{9 \times 8} = \frac{30}{54} \times \frac{30}{48}\right], \text{ fig. 103,}$$

we have no striking difference from the usual character of ordinary circulating curves, internal or external. As the ratio diminishes, the course of the curve shows less of the elliptic character, and more of the circular; till, when the ratio becomes one of equality,

$$\frac{25}{25}, \left[\frac{xx'}{yy'} = \frac{1 \times 2}{2 \times 3} = \frac{30}{60} \times \frac{32}{48}\right]$$

we obtain, with the adjustment for external loops, a perfect circle; and one circle only, without repetition.

For we have now arrived at the point where V = 1, and where n consequently = 0 when the epicycle is retrograde : a result already exemplified in fig. 5, page 16.

Fig. 102.

Fig. 101.

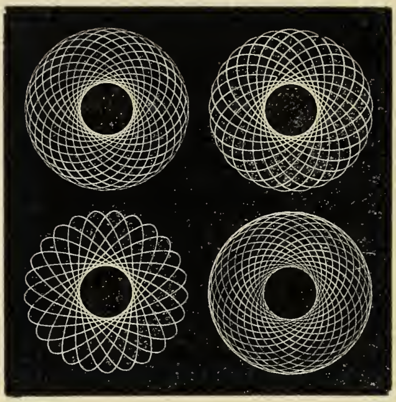


Fig. 100.

Fig. 103.

Now, all fractional values for V, occurring between the integers 2 and 1, fall within the special conditions referred to in the theoretical explanation quoted at page 19; where it appeared that if the velocity of the deferent (Flange) were greater than that of the epicycle (Frame), n would have to be replaced by $\frac{1}{n}$, and the resulting curve would be identical with that obtained when the velocities of the deferent and epicycle were exchanged, and n expressed their ratio. To take an illustration from the *external* varieties of the figure with 7 loops, for producing which some of the values for $\frac{xx'}{yy'}$ were discussed at page 135; we find the following duplicate cases :—

when $V = \frac{7}{2}$, $n = \frac{5}{2}$: and, when $V = \frac{7}{5}$, $n = \frac{2}{5}$.

(n of the latter $= \frac{1}{n}$ of the former) . . . (i)

The negative sign of n is disregarded, being common to both, and indicating direction of motion only. And, if the value given to a in the one case be used for b in the other, and that given to b in the former be used for a in the latter, the two curves are indistinguishable. Again,

$$\text{when } V = \tfrac{7}{3},\ n = \tfrac{4}{3}: \text{ and, when } V = \tfrac{7}{4}, n = \tfrac{3}{4}$$
$$\left(n \text{ of the latter} = \frac{1}{n} \text{ of the former}\right) \quad . \quad . \quad . \qquad \text{(ii)}$$

also,

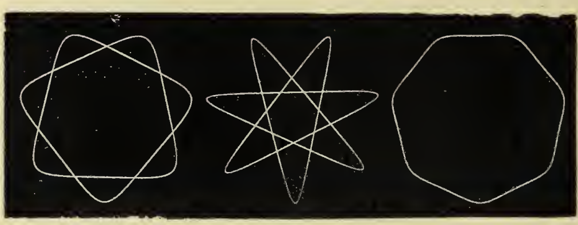
$$\text{when } V = \tfrac{7}{6},\ n = \tfrac{1}{6}: \text{ and, when } V = 7,\ n = \tfrac{6}{1}$$
$$\left(n \text{ of the latter} = \frac{1}{n} \text{ of the former}\right) \quad . \quad . \quad . \qquad \text{(iii)}$$

the same curves precisely are obtained, for each reciprocal pair, by the transposition of a and b.

To make this practically evident, we may compare the polygonal forms $\left(\frac{a}{b} = n^2\right)$ for each of these six values for V; and three of the figures will be found to correspond in all respects with the other three. Thus,

$$\text{(i)} \left\{\begin{array}{l} V = \dfrac{7}{2},\ n^2 = \dfrac{25}{4} = \dfrac{a}{b}: \text{ and, if } a + b = 45, \\ \quad a = 38{\cdot}8,\ b = 6{\cdot}2 \ \left[\dfrac{x'}{y'} = \dfrac{42}{36},\ x = y\right] \\ V = \dfrac{7}{5},\ n^2 = \dfrac{4}{25} = \dfrac{a}{b}: \text{ and, if } a + b = 45, \\ \quad a = 6{\cdot}2,\ b = 38{\cdot}8 \ \left[\dfrac{xx'}{yy'} = \dfrac{42}{54} \times \dfrac{30}{50}\right] \end{array}\right\} \text{fig. 104.}$$

$$\text{(ii)}\left\{\begin{array}{l} V = \frac{7}{3},\ n^2 = \frac{16}{9} = \frac{a}{b}: \text{ and, if } a + b = 45, \\ \qquad a = 28{\cdot}8,\ b = 16{\cdot}2 \left[\frac{x'}{y'} = \frac{42}{54},\ x = y\right] \\ V = \frac{7}{4},\ n^2 = \frac{9}{16} = \frac{a}{b}: \text{ and, if } a + b = 45, \\ \qquad a = 16{\cdot}2,\ b = 28{\cdot}8 \left[\frac{xx'}{yy'} = \frac{42}{54} \times \frac{30}{48}\right] \end{array}\right\} \text{fig. 105.}$$



(i) Fig. 104. (ii) Fig. 105. (iii) Fig. 106.

$$\text{(iii)}\left\{\begin{array}{l} V = \frac{7}{6},\ n^2 = \frac{1}{36} = \frac{a}{b}: \text{ and, if } a + b = 45, \\ \qquad a = 1{\cdot}2,\ b = 43{\cdot}8 \left[\frac{xx'}{yy'} = \frac{42}{54} \times \frac{30}{60}\right] \\ V = 7,\ n^2 = \frac{36}{1} = \frac{a}{b}: \text{ and, if } a + b = 45, \\ \qquad a = 43{\cdot}8,\ b = 1{\cdot}2 \left[\frac{xx'}{yy'} = \frac{42}{30} \times \frac{50}{30}\right] \end{array}\right\} \text{fig. 106.}$$

From these figures, and from further experiments of the same kind, it will be evident that whether V = N (where N is any integer) or $= \frac{N}{N - 1}$, the result is the same, when the epicycle is retrograde, *provided* (*a*) *and* (*b*) *are transposed.*

But if this be so, the change-wheels used for the

example where V = $\frac{25}{24}$, fig. 103, making $n = \frac{1}{24}$, *should afford a curve of* 25 "*consecutive*" *loops outwards*, provided (b) the excentricity of the Frame becomes so much larger than (a) the excentricity of the Flange as to disentangle the loops and exhibit them individually.

Fig. 107.

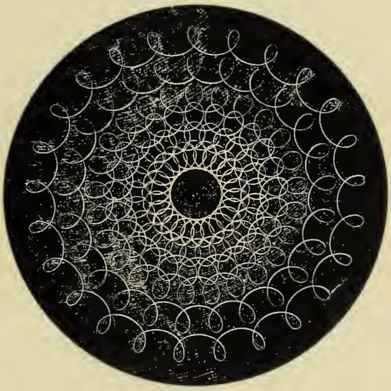


Fig. 107 proves that this deduction is fully warranted by the facts. The change-wheels are those ($x = 30$, $y = 48$, $x' = 30$, $y' = 54$) giving to V and n the values just stated : a was taken at 7 divisions only, for all the curves, while b was 80, 65, 50, 40, 30, and 20, in succession.

The behaviour of the instrument is now different altogether. The Frame revolves once, slowly, during the time in which the pulley makes 24 rotations, and traces the consecutive loops with much deliberation.

We saw at the commencement that when n could be expressed by $\frac{p}{q}$, the epicycle being retrograde, there would be $(p + q)$ apocentres and as many pericentres ; and accordingly we now find that where $n = \frac{1}{24}$, there are $(1 + 24 =)$ 25 loops produced. Similarly, in all

other cases, wherever V can be represented by a fraction whose numerator exceeds the denominator by unity, we may obtain consecutive loops equal in number to the numerator of that fraction. And, by giving to b a much larger value than to a—or rather, by considering the Frame to be now the "deferent," and the Flange the "epicycle," and assigning their radii accordingly—this transformation of the curve will be easily recognized. Therefore, in order to describe a curve *with any number* N *of consecutive external loops*, it will be only necessary to select four change-wheels which satisfy the simple equation

$$\frac{xx'}{yy'} = \frac{\text{N}}{3\,(\text{N} - 1)}\,*$$

This conclusion, which may be confirmed practically to any extent, will no doubt be as gratifying as it was, probably, unexpected, for it opens up an entirely new field of adjustment by placing at our command consecutive loops to almost any desired extent; and, subject to the limitation of depth of cut, far exceeding in their particular class, both in number and variety, any requirements for ornamental purposes.

Under the present system, which, as depending upon $\frac{1}{n}$ in place of n, may be called the "*reciprocal system*," it might be more correct to use b for the excentricity of the Flange, and a for that of the Frame, since the former now represents the epicycle and the latter the deferent. To avoid confusion, however, a and b will continue to be used in the same sense as hitherto: the former referring to the Flange and the latter to the Frame.

* It will be remembered that 3 is the multiplying effect of the permanent wheels of the train.

An alteration in the position of the tool-box upon the Frame seems now to repeat almost the same curve, as may have been already described, upon the circumference of a circle concentric with that upon which the previous curve may be supposed to stand: and the loops which correspond to one another are symmetrically placed on the same radii, since there has been no movement of the Flange to disturb them. But when the excentricity of the latter is varied in order to increase or diminish the size of the loops themselves, the distortion which has formerly occurred will again be present, and will need counteraction as before at the Tangent-wheel. And the formula deduced in the last chapter for the condition of two pairs of change-wheels, with both "carriers" in use,

$$\mathbf{C}_2 = 4 + \frac{2}{\text{V}}$$

is equally applicable for our present purpose.

But it is an essential part of this method of describing "consecutive" loops that V shall be very nearly equal to unity; and therefore that the fraction $\frac{yy'}{xx'}$ shall be very nearly equal to $\frac{1}{3}$. The greater the number of loops, the more nearly these values are approached; and even with comparatively low numbers, the error caused by regarding $\frac{yy'}{xx'}$ (for purposes of compensation) as equal to $\frac{1}{3}$, is less than will be found to arise from the inevitable errors of construction and adjustment.

This being granted, the equation now quoted becomes simply,

$$\mathbf{C}_2 = 4 + \tfrac{2}{1} = 6\,;$$

that is to say, the correction at the Tangent-wheel for

all curves described "reciprocally" may be considered to be irrespective of the number of loops and of the excentricities employed, and as equivalent to six divisions of the micrometer screw of the Tangent-wheel for one of the Flange. And it is some satisfaction to find that, as another variety of compensation is required, it proves to be so simple in its application.

Fig. 108.

Fig. 108 illustrates, with some attempt at decorative arrangement, the class of curves which, by the substitution of $\frac{1}{n}$ for n, have been brought within the compass of the Epicycloidal Cutting Frame with two removable arbors.

The change-wheels for the cusped border, and for the adjoining loops, were $\frac{x}{y} = \frac{30}{54}, \frac{x'}{y'} = \frac{30}{48}$; the former pair being placed upon the original arbor A next to the Eccentric Frame, and the latter upon B, which is carried by the radial plate. Since $n = \frac{1}{24} = \frac{a}{b}$ for the cusped condition, it is not practicable to describe three perfect cusps in such close proximity: the middle

one is taken of the exact ratio, and those on either side do not differ from it materially.

a	=	$3\frac{3}{4}$,	$3\frac{3}{4}$,	$3\frac{3}{4}$	=	4,	7,	10,	13
b	=	$92\frac{1}{2}$,	90,	$87\frac{1}{2}$	=	70,	70,	70,	70
c_2	=	0	0	0	=	24,	42,	1 10,	1 28

The series of four curves, each with 16 consecutive loops (the outer ones in contact, and all having $(a - b)$ constant) required the wheels

$$\frac{x}{y} = \frac{36}{60}, \frac{x'}{y'} = \frac{32}{54}:$$

$c_2 = 6$, as in all similar cases.

a	=	6,	8,	10,	12
b	=	34,	36,	38,	40
c_2	=	36,	48,	1 10,	1 22

The circulating figure in the centre of the design belongs to the instrument in its simple form; it is a curve of 16 loops, and the wheels given in Table III. are $\frac{32}{42}$. These were taken for x' and y', a "blank" being placed between them, and a single 48 wheel, as "carrier," was placed upon the arbor A. In accordance with the remarks at the close of the last chapter, the compensation will be $c_2 = 4 + \frac{2y'}{3x'} = 4\frac{7}{8}$; and, though not needed for the inner line, which was made parallel to the first by altering b only, a correction for the *first* was necessary to ensure its symmetrical distribution with reference to the preceding group:—

$$a = 10; \ b = 15 \text{ and } 17\tfrac{1}{2}; \ c_2 = 49,$$

the "initial position" having been carefully verified in all the cases.

The following short table gives, with their calculations, the consecutive loops which can be obtained, on what has been called the "reciprocal system," from fifteen change-wheels, comprising the original set of twelve, and the three others (54, 50, and another 30) already named in connection with the additional arbor. Though not numerous, the list is not deficient in range or variety, and offers, without increasing the assortment of wheels, wide scope for ingenious design.

TABLE VI.

Loops	$\frac{V}{3}$	Arbor A x	y	Arbor B x'	y'
12	$\frac{4}{11} = \frac{1}{2} \times \frac{8}{11}$	32	60	30	44
15	$\frac{5}{14} = \frac{3 \times 5}{7 \times 6}$	30	60	30	42
16	$\frac{16}{15 \times 3} = \frac{2 \times 8}{3 \times 15}$	36	60	32	54
17	$\frac{17}{16 \times 3} = \frac{1}{2} \times \frac{34}{48}$	34	60	30	48
24	$\frac{8}{23} = \frac{1}{2} \times \frac{16}{23}$	32	60	30	46
25	$\frac{25}{24 \times 3} = \frac{5 \times 5}{9 \times 8}$	30	54	30	48
45	$\frac{15}{44} = \frac{1}{2} \times \frac{30}{44}$	30	60	30	44
51	$\frac{17}{50} = \frac{1}{2} \times \frac{34}{50}$	34	60	30	50

To develope, however, this principle as fully as the present extension will permit, the set of change-wheels should comprehend the whole of the numbers, both even and uneven, from 30 to 60. Also, since their effect is, in all these cases, to retard and not to accelerate—the small wheels driving the larger ones, and not *vice versâ*—we may extend the list downwards from 30

to 24 inclusive. It is true that this may be a dangerous suggestion, as offering an inducement to try the effect of these small wheels conversely, bringing V up to 15 or more; but such a course is earnestly to be deprecated wherever respect is entertained for the condition of the instrument.

TABLE VII.

Change-wheels for producing consecutive EXTERNAL *Loops.**

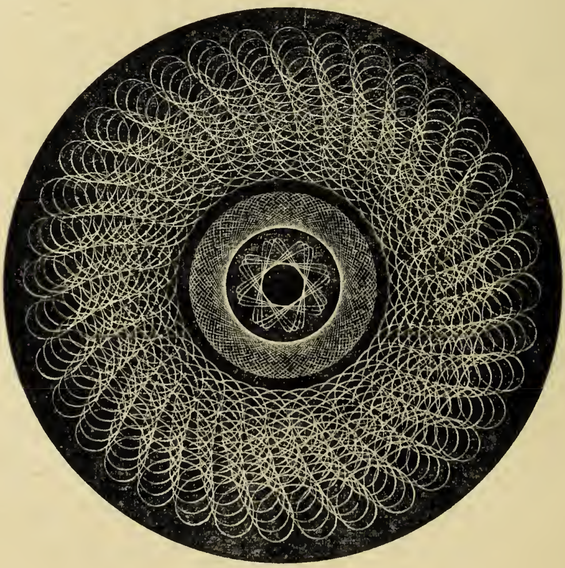
Loops	Arbor A x y	Arbor B x' y'	Loops	Arbor A x y	Arbor B x' y'	Loops	Arbor A x y	Arbor B x' y'
11	30 60	33 45	34	34 54	24 44	65	35 56	26 48
12	30 60	32 44	35	35 60	30 51	66	44 60	24 54
13	30 60	39 54	36	32 56	30 50	70	30 54	28 46
14	32 60	35 52	37	37 60	30 54	75	30 60	25 37
15	30 60	30 42	39	39 60	30 57	76	38 60	24 45
16	36 60	32 54	40	40 54	24 52	77	35 60	33 57
17	34 60	30 48	41	41 60	28 56	81	30 60	27 40
18	36 60	30 51	42	35 60	24 41	85	34 60	25 42
19	38 60	30 54	44	33 54	24 43	86	43 60	24 51
20	32 57	30 48	45	30 60	30 44	87	30 60	29 43
21	35 60	30 50	46	46 60	24 54	93	31 60	30 46
22	33 56	32 54	48	32 60	30 47	95	38 60	25 47
23	46 60	25 55	50	32 49	25 48	96	48 60	24 57
24	32 60	30 46	51	34 60	30 50	99	33 60	30 49
25	30 54	30 48	52	32 51	26 48	105	35 60	30 52
26	32 50	26 48	54	36 60	30 53	111	37 60	30 55
27	36 60	30 52	55	33 54	25 45	115	46 60	25 57
28	32 54	28 48	56	32 55	28 48	117	39 60	30 58
29	30 60	29 42	57	38 60	30 56	124	31 54	24 41
30	40 60	30 58	58	32 57	29 48	126	42 60	44 50
31	30 60	31 45	60	40 60	30 59	130	30 54	26 43
33	33 60	30 48	64	32 54	24 42	141	47 60	24 56

Table VII. gives a full selection of consecutive loops which would be then attainable, commencing with eleven, the point where they were discontinued in Table V. The choice of numbers of loops is much

* As to distribution of the change wheels, see p. 133.

greater than can be practically requisite; and, if it be preferred, the series of additional change-wheels may be materially abridged without greatly interfering with their convenient application. The value for V is nearly uniform throughout, being never less than 1, and never greater than 1·1. Any driving velocity, therefore, which is satisfactory in one case will be equally so in all; and there is now less risk of over-driving the instrument; indeed, under present arrangements, the pulley must rotate pretty smartly, to communicate a suitable speed to the cutting tool. And, for the same reason, the preliminary adjustments for the mutual position of Flange and Frame are more readily effected, while an error in applying the correction at the Tangent-wheel is of less importance.

Fig. 109.

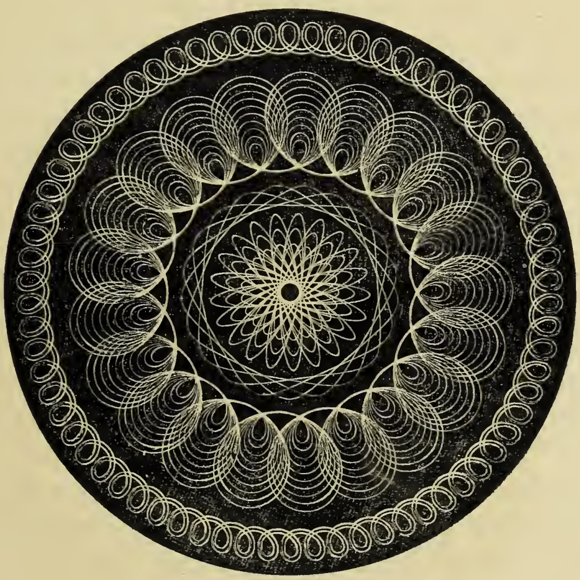


The deep border of fig. 109 consists of a series

of 36 loops, arranged for consecutive contact at the outer circumference, and continued until alternate contact takes place. The excentricity of the Flange was uniformly 15 divisions; and that of the Frame was diminished from 120 to 70 by intervals of 5. To obtain a spiral effect, the micrometer screw of the Tangent-wheel was moved 20 divisions between each two curves. The origin of the two interior figures is sufficiently obvious without explanation.

In the next figure (fig. 110) the border is introduced rather as a warning than as an example. It is unreasonable to expect from an Epicyclic train with so

Fig. 110.



small a terminal wheel as the 40 on the axis of the Eccentric Frame, accurate delineation of loops of small dimensions, when the values for *a* and *b* are so dis-

cordant. If small loops are desired, the circle upon whose circumference they are placed should not be a large one; and, when the design extends to some distance from the centre, the exterior figure should be composed of loops of fair size. The border in question contains 70 loops, described with the highest value (134) for b which the Frame admits: a was only 6 and 8 divisions of the Flange for the inner and outer curves respectively; and, consistently with the result obtained above for the compensation in such cases ($\mathbf{c}_2 = 6$), the correction of 12 divisions at the Tangent-wheel was applied for the second curve.

The series of eleven curves, each with 21 loops, had $(a - b)$ constant for all: a was increased by two divisions at a time from 8 to 28 inclusive, and b extended similarly from 72 to 92. The same correction of six divisions at the Tangent-wheel for each division on the Flange was continued; and, though the loops are of somewhat large dimensions, the result, as regards correctness of compensation, is fairly satisfactory.

Interior to these loops is a curve, of the same number (21), obtained with $\frac{x'}{y'} = \frac{42}{30}$, and a single wheel connecting x' with the 40 on the Eccentric Frame. a was $51\frac{1}{4}$ and b 5, their ratio being equal to n^2, and the curve partaking in consequence of a right-lined character.

As a matter of symmetry, the central star has also 21 foliations; and, there being no appropriate combination to be found in Table III., change-wheels were calculated for the desired value

$$V = \frac{21}{10}, \quad \frac{xx'}{yy'} = \frac{7 \times 3}{6 \times 5}, \quad \frac{x}{y} = \frac{42}{50}; \quad \frac{x'}{y'} = \frac{30}{36}$$

which produce the features of the ellipse. The curve is not "reciprocal," and the compensation here is

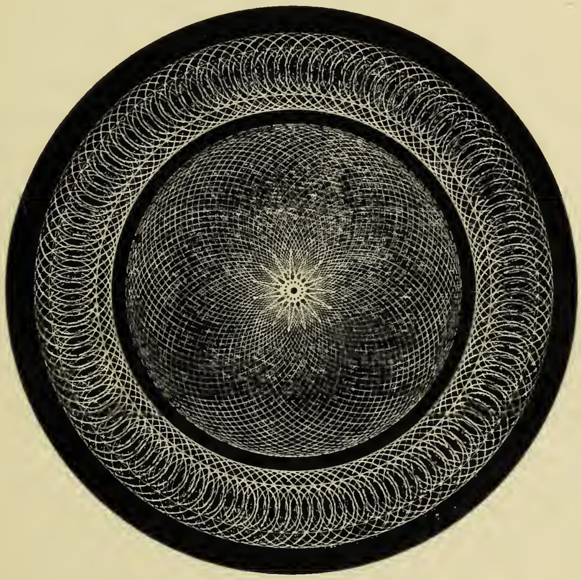
$$\mathbf{C}_2 = 4 + \frac{2}{V} = 4 + \frac{2}{1} \times \frac{10}{21} = \frac{20}{21} = 5 \text{ nearly};$$

and the adjustments were

a	=	10,	14,	18
b	=	15,	19,	23
$\mathbf{C}_2$	=	1 0,	1 20,	1 40

Figure 111 shows that loops of high numbers may be occasionally useful. The border consists of two

Fig. 111.



curves, each containing 126 consecutive loops, where b was equal to 115, and a to $12\frac{1}{2}$ and 20. The "engine-turned" centre is a curve of 85 loops described by one

revolution of the Frame ($a = 47$, $b = 43$), but no advantage is apparent in favour of this method compared with an ordinary circulating curve such as is shown in fig. 51, unless the evidently more circular course of the curve be so considered.

It will have been observed that no mention has been made of consecutive *internal* loops obtained on this "reciprocal" system; and it might be expected that a withdrawal of the second "carrier" would suffice, as on other occasions, to change the direction of the curve while preserving its characteristics. But, on making this alteration in the direction of motion, the individuality of the loops disappears altogether, the revolution of the Frame axis becomes much more rapid, and the result is a crowded circulating curve of no interest. The reason of this is plain. When V is fractional, and slightly in excess of unity, and the epicycle is retrograde, n (which $= V - 1$) is negative, and a very small quantity indeed. But when, with a value of this kind for V, the epicycle becomes direct, n (which is then $= V + 1$) acquires a value which may be a hundred times or more the value of n in the former case. This is a change so much greater than any which occurs when the negative value of n exceeds unity, that no relationship appears to subsist between curves described with the same value for V but such widely different values for n.

But, strange as it may appear, it is possible to reverse the direction in which the Frame has been moving with reference to the Flange—and, consequently, to make the epicycle direct and the loops internal—upon a different principle, and without changing the number of axes in the train. The description of the method by which this is accomplished is given in the following chapter.

CHAPTER X.

CURVES WITH CONSECUTIVE INTERNAL LOOPS OF HIGH NUMBERS SIMILARLY PRODUCED. — "RECIPROCAL" CIRCULATING CURVES.

WHEN describing, in the manner explained in the last chapter, curves with consecutive external loops of high numbers, it has been shown that the intrinsic or numerical value for V was always slightly in excess of unity; and that the higher the number of loops, the more nearly did V become equal to that quantity. And, if the method has been examined experimentally, it will have been also remarked that the extreme slowness of the motion of the Frame, compared with that of the Flange, has been the more decided, the greater the number of loops—i. e., the more nearly V became equal to unity. But so long as V remains an improper fraction, the revolution of the Frame, however slow, is still in the direction opposed to that of the Flange: the epicycle is therefore "retrograde," and the loops consequently external.

Now, while the two "carriers" both form part of the train, let change-wheels be selected, and placed upon their proper arbors in the instrument, which will make $\frac{xx'}{yy'} = \frac{1}{3}$, and therefore V *equal* to unity: and let the number of variable axes in the train be the same as before, viz., two "carriers," and two removable arbors. A singular curiosity in epitrochoidal motion is now

apparent; for the rotation of the Frame, slow as it was under the former conditions, will be found to cease altogether. If the Flange be at all excentric, the Frame will move, remaining parallel to its assigned position at the commencement, but having no axial rotation: and the point of the tool will describe a circle whose radius is equal to the excentricity which the Flange may possess, and whose locality is determined by the excentricity of the Frame and the position of the zero point on the Tangent-wheel.* But if the Flange be strictly central, the Frame loses even this motion of translation, and remains absolutely at rest, however rapid and numerous may be the rotations of the pulley.

Next, both "carriers" being still employed, let change-wheels be taken—as they can be without difficulty, since there are two arbors for their accommodation—which will give to $\frac{xx'}{yy'}$, a value very little *less* than $\frac{1}{3}$. We shall now see that the Frame, though moving as slowly as formerly, turns in *the same* direction as the Flange: the epicycle has therefore become "direct," and the loops, when formed, will be internal.

Referring to what was said on page 72, when considering the effect of the short train from S to M only, that portion of the instrument was proved to be an apt illustration of the mechanical puzzle known as "Ferguson's Paradox"; for we saw that, during the radial adjustment of the Flange, the Frame acquired a motion of rotation to the right, or to the left, or none at all—according as the value of that short train happened to be greater than, less than, or equal to, unity. And

* Fig. 5, suprà.

this similarity to the contrivance in question continues when the instrument is considered with reference to the whole train.

The three cases may be stated briefly as follows, in the form of an example already adopted: and it will be observed that, while three of the change-wheels remain the same, the addition of two teeth to the fourth wheel serves to obliterate the previous external loops, and a second increase of two teeth in the same wheel to restore them, equal in number, but opposed in direction, as compared with the first:—

(i) $x = 30$, $y = 54$, $x' = 30$, $y' = 48$. $V = \frac{25}{24}$, $n = \frac{1}{24}$; the Flange and Frame revolve in opposite directions: the epicycle is therefore "retrograde," and the loops external.

(ii) $x = 30$, $y = 54$, $x' = 30$, $y' = 50$, $V = 1$, $n = 0$; the Frame does not revolve at all, and there is no motion in the epicycle.

(iii) $x = 30$, $y = 54$, $x' = 30$, $y' = 52$, $V = \frac{25}{26}$, $n = \frac{1}{26}$; the Flange and Frame revolve in the same direction: the epicycle is therefore "direct," and the loops are internal.

The number of axes in the train is the same in the three cases; and in all, the numerical value of n is *less by* 1 than that of V.

In the last chapter, by putting $\frac{1}{n}$ for n, and transposing a and b, consecutive *external* loops were obtained equal to those which would be produced where n had not been inverted, nor a and b interchanged; so now, by the same "reciprocal" treatment, we ought to be able to describe *internal* consecutive loops with the same facility. Since also, when the epicycle is direct, and

$n = \frac{p}{q}$, there are* $(p - q)$ or $(q - p)$ apocentres, and as many pericentres, the number of internal loops should be $(26 - 1 =)$ 25, with the change-wheels, and relative adjustments of a and b, which are now proposed.

Fig. 112 is a satisfactory proof of the correctness of this assumption: it is precisely the counterpart of

Fig. 112.

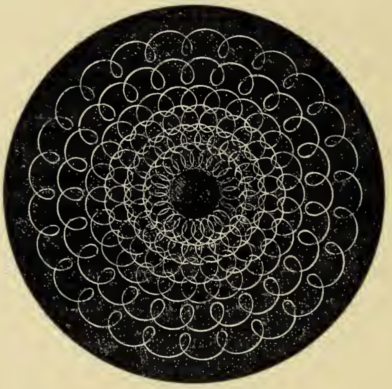


fig. 107, the excentricities adopted for a and b respectively being the same as in that figure, and the change-wheels employed being those stated in case (iii) above.

Therefore, in order to describe a curve *with any number*, N, *of consecutive internal loops*, it will be only necessary to select change-wheels satisfying the simple equation $\frac{xx'}{yy'}, = \frac{N}{3(N + 1)}$.†

The range of numbers from which a selection of consecutive internal loops may be made is as extensive as in the previous, and analogous, case which formed the subject of the last chapter; though, as regards the

* Art. "Trochoidal Curves," Penny Cyclopædia. † Compare p. 160.

three additional wheels specially considered in Table VI., the facilities do not appear to be quite so numerous. Table VIII. has been calculated in the same manner as Table VII., and within the same limits.

TABLE VIII.

Change-wheels for producing consecutive INTERNAL *Loops.**

Loops	Arbor A x y	Arbor B x' y'	Loops	Arbor A x y	Arbor B x' y'	Loops	Arbor A x y	Arbor B x' y'
11	33 60	30 54	36	30 60	24 37	77	33 54	28 52
12	32 60	30 52	37	37 60	30 57	81	30 60	27 41
13	30 60	26 42	38	38 54	24 52	84	30 50	28 51
14	36 60	28 54	39	30 60	26 40	85	34 60	25 43
15	30 60	30 48	40	30 54	24 41	86	43 58	24 54
16	32 60	30 51	42	30 60	28 43	87	30 60	29 44
17	34 60	30 54	44	44 60	24 54	90	32 56	30 52
18	30 57	30 50	45	30 60	30 46	91	35 60	26 48
19	38 60	28 56	48	32 60	30 49	93	30 60	31 47
20	32 56	30 54	50	30 54	30 51	94	47 60	24 57
21	30 60	28 44	51	34 60	30 52	95	38 60	25 48
22	33 54	24 46	52	36 54	26 53	99	33 60	30 50
24	32 60	30 50	54	36 60	30 55	104	26 54	24 35
25	30 54	30 52	55	44 60	25 56	105	35 60	30 53
26	32 54	26 48	56	42 57	24 54	111	37 60	30 56
27	36 56	24 48	57	38 60	30 58	114	38 60	24 46
28	36 58	28 54	58	36 59	29 54	115	46 60	25 58
29	30 60	29 45	62	31 54	24 42	116	29 54	24 39
30	25 60	24 31	64	32 52	24 45	117	39 60	30 59
31	31 60	30 48	65	30 54	26 44	120	40 55	32 44
32	32 55	30 54	68	34 54	24 46	124	41 60	24 50
33	33 60	30 51	69	46 60	24 56	125	30 54	25 43
34	34 60	24 42	74	37 60	24 45	128	32 54	24 43
35	42 60	25 54	75	35 57	30 56	129	43 60	24 52

It would be perfectly easy to treat the lower numbers, from 10 downwards, by the same reciprocal method, both externally and internally: and it is very probable that the moderate velocity which the Frame would then acquire may be generally considered preferable—especially for values of V between 6 and

* As to distribution of the change-wheels, see p. 133.

10—to the higher speed which is characteristic of the double-arbor extension, as first described in Chapter VIII.

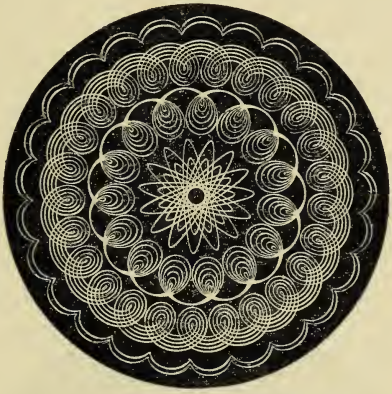
The Tables have been continued at least as far as will be necessary, but the system is susceptible of an extension terminated only by the selection of change-wheels with which the instrument may be provided; and some numbers of considerable magnitude are within the compass of the set which has been lately assumed. For instance, 231 internal loops are available; because the fraction $\frac{231}{232}$, when divided by 3, can be expressed as $\frac{33 \times 28}{58 \times 48}$, and wheels of those numbers can be placed upon the proper arbors. But the same number of external loops does not seem to be practicable, as the fraction $\frac{231}{230}$ is not so accommodating, and a wheel either of 22 teeth or 69 would be needed. The number 351, however, could be used for loops of either direction, since we have $\frac{351}{350} \times \frac{1}{3} = \frac{26}{50} \times \frac{36}{56}$; and $\frac{351}{352} \times \frac{1}{3} = \frac{26}{44} \times \frac{27}{48}$. At the same time, it is not easy to see that such combinations as these can possess any practical utility; except possibly for a variety in "grailing."

The symmetrical disposition of the loops in fig. 112 naturally corresponds to that in fig. 107, the Flange having been undisturbed in both; but former experience will have shown the necessity for a correction at the Tangent-wheel whenever an alteration has been made in the excentricity of the Flange. And, since no change has taken place in the number of axes in the train, we shall be justified in expecting that—by parity of reasoning to that in the subdivision of the second case of the investigation where one removable arbor was concerned—the compensation required will

be the same as we have last employed. That is, that we shall have $C_2 = \mathbf{C}_2 = 6$, when the loops are so numerous that V differs from unity by an inconsiderable amount.

Fig. 113 is offered as an appropriate companion to fig. 108, being composed of internal loops and cusps in similar order.

Fig. 113.



The cusped border, and adjoining series of loops, were described with the change-wheels lately mentioned, $[x = 30, y = 54, x' = 30, y' = 52]$, giving to n the value $\frac{1}{26}$, which also represents the proper ratio between a and b for the internal cusps. a was taken $= 3\frac{1}{2}$, therefore $b = 91$, and the border was doubled by a curve with the same value for a, and having $b = 88\frac{1}{2}$. For correction at the Tangent-wheel, $C_2 = 6$; and 21 divisions of the micrometer screw were needed for the $3\frac{1}{2}$ at the Flange.

The following were the adjustments for the next :—

a =	4,	6,	8,	10,	12
b =	70,	70,	70,	70,	70
C_2 =	24,	36,	48,	1 10,	1 22

Internal loops require more room, as it were, than those which are external : and the next group consists of fifteen, instead of sixteen, which were adopted in the corresponding figure [x = 30, y = 60, x' = 30, y' = 48]

a	=	12^{1}.	11,	$9\frac{1}{2}$,	8,	$6\frac{1}{2}$,	5
b	=	$44\frac{1}{2}$.	46,	$47\frac{1}{2}$,	49,	$50\frac{1}{2}$,	52
C_2	=	1 25,	1 16,	1 7,	48,	39,	30

The double curve in the centre of the figure is one of those requiring one pair of change-wheels only [x' = 30, y' = 48]; a was 8 and 12; b, 12 and 18 successively. Both arbors were retained, the first carrying only a single wheel.

Fig. 114.

Fig. 114 may serve to show that the external and internal forms of curves obtained "reciprocally" can be combined as readily as those of more simple origin ;

and, since the two "carriers" remain in action under both circumstances, there are now fewer adjustments to be made when reversing the direction of the loops. The numbers of the loops to be employed for any design may also be so arranged as to require only slight modifications of the train. In the present figure, for example, x, y, and x', were undisturbed throughout; and the change of y' from 44 to 46, 52, and 52, in succession, was sufficient to develope the 12 and the 24-looped figures in both varieties.

The construction of the figure hardly needs explanation, beyond the remark that, to effect the correspondence of the doubled 24-looped curve in each form, the Tangent-wheel was moved, for either of the two, through the $\frac{1}{48}$ part of the circumference (i. e. 2 turns of the Tangent-screw), after the usual preliminary correction and adjustments had been made. It will be noticed that the irregularities, consequent upon the large excentricity of the Frame, and the small size of the wheel upon its axis, are less apparent in the internal form of the curve than in the external.

What has been termed the "reciprocal system" is not confined to those fractional values for V whose numerator and denominator differ by unity. Any two numbers whatever, which are prime to one another, and are not widely different in magnitude, may constitute the fraction; and if, when divided by 3 (the multiplying effect of the permanent wheels of the train), that fraction can be expressed in terms of four of the given change-wheels, we shall obtain a "circulating" curve whose loops are equal in number to the greater of the two numbers, and which will take as many revolutions of the Frame for its completion as are equal to their difference. The loops, moreover, will be internal or

external, according as V is a proper or an improper fraction.

For example, if we take the numbers 25 and 21 :—

(i) Let $V = \frac{25}{21}$: then

$$n = \frac{4}{21}, \text{ and } \frac{xx'}{yy'} = \frac{5 \times 5}{7 \times 9} = \frac{30}{42} \times \frac{30}{54}:$$

and, using *both* "carriers," with a larger excentricity for the Frame than for the Flange, 25 (= 21 + 4) "circulating" *external* loops are produced.

(ii) Let $V = \frac{21}{25}$: then

$$n = \frac{4}{25}, \text{ and } \frac{xx'}{yy'} = \frac{7 \times 2}{10 \times 5} = \frac{24}{60} \times \frac{35}{50}:$$

retaining *both* "carriers," and a similar proportion between *a* and *b*, 21 (= 25 − 4) "circulating" *internal* loops are the result.

In both cases four revolutions of the Frame will complete the curve.

The concluding figure (fig. 115) illustrates the adaptability of the method here indicated; which, from the more convenient velocity ratio between Flange and Frame, and from the greater freedom of movement now permitted to the cutting-tool, is in every way an acquisition.

In the border of external loops the fractional value for V is $\frac{35}{32}$, therefore $n = \frac{3}{32}$, and 35 loops are described, outwards, in three revolutions of the Frame: the wheels required are

$$\frac{xx'}{yy'} = \frac{1}{3} \times \frac{35}{32} = \frac{35}{48} \times \frac{30}{60}:$$

and, if the internal form of the same curve had been introduced, we should have had V $= \frac{35}{38}$, and $n = \frac{3}{38}$; the wheels might have been arranged as

$$\frac{xx'}{yy'} = \frac{35}{57} \times \frac{30}{60};$$

and the 35 internal loops would be completed in like manner by three revolutions of the Frame.

In this example, the compensation was obtained by

Fig. 115.

the same formula which has been proved experimentally to hold good for all values of xx', yy', when both "carriers" are employed. $\frac{2}{\mathrm{V}}$ is here $= \frac{64}{35} = 1{\cdot}8$: therefore $\mathbf{C}_2 = 4 + \frac{2}{\mathrm{V}} = 5{\cdot}8$. The adjustments were:—

a	=	10,	$12\frac{1}{2}$,	15
b	=	65,	$67\frac{1}{2}$,	70
$\mathbf{C}_2$	=	1 8,	1 $22\frac{1}{2}$,	1 37;

the quantities stated in the last line being increased, as in all similar cases, by the zero point of the Tangent-wheel for the time being.

The intermediate curve is one of 27 loops, which occur alternately, and are obtained by assigning to V the value $\frac{27}{29}$, i. e. $n = \frac{2}{29}$. The wheels were

$$\frac{xx'}{yy'} = \frac{36}{58} \times \frac{30}{60};$$

and, since for all curves produced "reciprocally"

$$C_2 = \mathbf{C}_2 = 4 + \frac{2}{V};\text{—and here, } \frac{2}{V} = \frac{58}{27} = 2{\cdot}15,$$

the compensation needed was $C_2 = 6{\cdot}15$: b was taken at 40 for both curves, and a was first 5, and then increased to $7\frac{1}{2}$, the angular correction being accomplished between the two.

In adapting the instrument to the 27 internal, from the 35 external loops, it has only been necessary to change the wheels on the arbor B (paying proper attention to the "initial position"); and in moving from this curve to the next, the substitution of a 42 for the 58 upon that arbor is all that is required to change the number and direction of the loops in the manner represented.

The central figure is a simple curve of 9 loops: it was described "reciprocally," with

$$V = \frac{9}{7}, \quad n = \frac{2}{7}, \quad \frac{xx'}{yy'} = \frac{36}{42}$$

as the more convenient course, owing to the previous arrangements of the removable arbors, but differs in no respect from the curve given by the single pair of change-wheels $\frac{x}{y} = \frac{60}{40}$, when $V = \frac{9}{2}$, and $n = \frac{7}{2}$.

There is a very serious objection to this method of obtaining loops of high numbers, arising from the

manner in which the tool is presented to the work. In the more usual disposition of the moving parts, when the Frame goes faster than the Flange, the course of the tool, though not quite uniform, is such that the plane of its upper surface is generally at right angles to the path which it is pursuing. But if the curve be described reciprocally, by the interchange of the velocities and excentricities of Flange and Frame, the angle at which the cutting-edge meets the work is continually varying. The tool has sometimes to cut sideways, and sometimes even backwards: and, in the latter position, the resistance which it necessarily encounters is so considerable that (the mechanism being not altogether devoid of elasticity) it is apt to spring away from its proper course upon returning to a more favourable inclination. This defect in the system, which increases with the number of loops, practically limits the application of the method to a cutting-tool of a very acute angle, and to a depth of penetration *not exceeding the hundredth of an inch.* The graduations of the Goniostat—an inevitable accompaniment in some form—extend to about 58°; and, if the "double-angled" tool be ground to that extreme degree, the section of its point will be nearly that of an equilateral triangle, and it will work pretty smoothly in all positions.

When the advantages of the "reciprocal system," as described in this and the preceding chapter, were mentioned to Mr. Pomeroy, he pointed out that its application would be within the compass of the instrument in the simple form in which he had designed it. For, by adding the requisite wheels—extending the series upwards to 72, as it has already been supposed to be carried downwards to 24—the following external loops

can be obtained with the single removable arbor, the first "carrier" being used without the second.

Loops.	1,	7,	8,	9,	13,	15,	18,	24,	25,	36,	72
x =	24,	28,	24,	27,	26,	25,	24,	24,	25,	24,	24
y =	72,	72,	63,	72,	72,	70,	68,	69,	72,	70,	71

Internal loops, however, are not so easily achieved, owing to the differing form of the fraction whose value has to be expressed by $\frac{x}{y}$. The one-looped figure will be produced in the internal form by the wheels named above, $x = 24$, $y = 72$, if the second "carrier" be added to the train, because this is not a case of "reciprocal" motion. But it does not seem practicable, since 72 is the largest wheel which the removable arbor will receive, and 24 the smallest, to give to x and y any suitable values whatever, within those limits, which would afford internal loops reciprocally.

To meet this difficulty, and to render the adaptation of the general principle more easy, Mr. Pomeroy has treated the arrangement somewhat differently to that which has been recently discussed, and has provided for the exceptional reduction of the value of the permanent wheels of the train from 3 to 1 : so as to dispense with the second removable arbor and its radial plate. For this temporary purpose, the two single "carriers" are replaced by two wheels upon one supplementary arbor, of which the larger (48) gears with the fixed central (64), and the smaller (30), upon the same axis, leads into the (60) wheel attached concentrically to the (32) upon the radial arbor of the instrument. The last-named wheel has thus no part in the train, and the (60), to which it is keyed, becomes merely a "carrier" to the rest : consequently the arrangement and value of the

whole train, after this modification, will be expressed as follows :—

$$\frac{64}{48} \times \frac{30}{y} \times \frac{x}{40} = V = \frac{x}{y}.$$

By this plan two axes less are required than when the double arbor extension is applied to this system. The change-wheels also for any given number of loops are limited to a single pair, and are nearly self-evident, without calculation. The number of consecutive loops attainable would not much exceed sixty, but that restriction is more than counterbalanced by the simplicity and elegance of the scheme.

At the same time, if any alteration is to be made in the normal value of the permanent wheels of the train, it may be worth while to embrace the opportunity thus presented of correcting, or at least diminishing, that irregularity of motion which is apparent when the loops are very small compared with their number, and which is exhibited by the border of fig. 110. This defect, as suggested in the description of that figure, is doubtless due in great measure to the comparatively small diameter of the (40) wheel upon the Frame axis; and it would probably be remedied almost entirely by the substitution of as large a wheel as the neighbouring parts of the mechanism will admit.

The disturbance of the Frame axis in its socket, which would be necessary for the occasional exchange of its terminal wheel, may be thought objectionable; but this was a feature of the original instrument which was invented by Captain Ash, and upon which the Epicycloidal Cutting Frame is founded. Moreover, whether the reduction of the multiplying effect of the permanent wheels of the train from 3 to 1 be attained

by either of these devices, or by any other, the arrangement might be considered perpetual; and all curves with consecutive loops exceeding three in number could be described "reciprocally." The principle remains the same: its application will be a matter partly of taste and partly of convenience.

When the value of the permanent wheels of the train has become equal to unity, so that $V = \frac{x}{y}$, x may in all cases be taken for the number of loops proposed; and the corresponding value for y will be such that $(x - y)$ for external loops, and $(y - x)$ for internal, shall be equal to the number of revolutions of the Frame required to complete the curve. For example, 36 consecutive loops would be obtained by adopting a wheel of that number of teeth for x; and either a 35 or a 37 for y, according to the external or internal character of the curve. Similarly, for figures of circulation, taking 37 for the number of loops, since 36 is not susceptible of that variety, and supposing that the curve will be such as to require two revolutions of the Frame for its description, we shall have $x = 37$; and $y = 35$ for external loops, or $y = 39$ for internal.

Therefore, if N = number of loops required, and R = number of revolutions of Frame axis in which it is intended that the curve shall be completed, we may say generally that, under these special circumstances, $\frac{x}{y} = \frac{N}{N \pm R}$. And, when the number of loops is so small that this fraction cannot, while in its lowest terms, be expressed by wheels of adequate size, its numerator and denominator may obviously be doubled or trebled at discretion: e. g. for 23 loops described at twice, 46

would be the number to select for x, and 42 or 50 for y, as the case may be.

The compensation will follow the laws which have been deduced in previous cases, and will depend upon the precise arrangement and number of the axes employed. The value of "the short train from S to M" will determine the angle of displacement for any given alteration at the Flange; and "the angle of correction" imparted to the Tangent-wheel will be V times less than that which is thence communicated to the axis of the Frame. Bearing these points in mind, and consulting, in case of need, one of the Treatises on the Principles of Mechanism to which reference has been made on former occasions, the amateur will be able to arrange a formula suited to the construction of instrument he may prefer or design.

For a more complete examination of these and similar conditions, together with greater range of adjustment, recourse may be had to the Geometric Chuck; which, though possessing in some respects superior facilities, will hardly be found so attractive and manageable, nor, if the excentric slide be rectilinear, so susceptible of theoretical compensation, as the excellent Epicycloidal Cutting Frame.

APPENDIX.

THERE can be no doubt that the mutual approach of the loops of the curve, whether internal or external, and their final contact, must depend upon fixed laws; and that the conditions for such contact can be stated mathematically. Since the ratio $\frac{a}{b}$ is the same when the loops touch, whether n is positive or negative, but of the same numerical value, (—for instance, six external loops, and four internal loops, equally attain their position of contact when $\frac{a}{b} = \frac{5}{4}$),—it seemed probable that some expression might be found, involving n only, to represent $\frac{a}{b}$ under the required conditions. After seeking in vain, both analytically and by methods of trial and error, for such an expression—which it is now evident must include the value of the "deferential angle" ϕ,—the writer of these Notes was fortunately able to submit the problem to the consideration of the Rev. H. W. Watson, M.A., Rector of Berkeswell and formerly Fellow of Trinity College, Cambridge, who has very kindly written for these pages the following explanation. It is founded upon the Treatise already so often cited, and the symbols have the same signification :—

"It is proved in the article 'Trochoidal Curves,' in the Penny Cyclopædia, page 289, that all the multiple points, that is, all the points through which more branches of the curve than one pass, must lie on either apocentral or pericentral radii.

"Hence it follows, that, if there be contact between any two loops, the points of contact must be situated upon apocentral

or pericentral radii, and that such radii must be tangents to each of the touching loops at the points of contact.

"In other words, where contact is possible, either an apocentral or pericentral radius may be found such that, if it be taken for the axis of x, the values of y and $\frac{dy}{dx}$ may vanish simultaneously.

"1°. If n be *positive* and the axis of x coincide with an *apocentral* radius, then in the article above quoted the equations of the curve are proved to be

$$x = a \cos \theta + b \cos n\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 1$$
$$y = a \sin \theta + b \sin n\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 2$$

"If a value of θ can be found such that y and $\frac{dy}{dx}$ vanish simultaneously, we have (representing this value by α)

$$a \sin \alpha + b \sin n\alpha = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad 1$$
$$a \cos \alpha + nb \cos n\alpha = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad 2$$
$$\therefore \tan n\alpha = n \tan \alpha \quad . \quad . \quad . \quad 3$$
$$\text{and } \frac{a^2}{b^2} = \frac{n^2 (1 + \tan^2 \alpha)}{n^2 \tan^2 \alpha + 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 4$$

"When n is given, $\tan \alpha$ may be found from 3, and then the ratio of $\frac{a}{b}$ may be obtained by substituting the value of $\tan \alpha$ in equation 4.

"2°. If (n) be *positive* and the axis of x coincide with a *pericentral* radius, we may easily prove that the equations are

$$x = a \cos \theta - b \cos n\theta$$
$$y = a \sin \theta - b \sin n\theta;$$

and, therefore, if a value (α) of θ can be found such that y and $\frac{dy}{dx}$ vanish simultaneously, we easily get

$$a \sin \alpha - b \sin n\alpha = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad 1'$$
$$a \cos \alpha - nb \cos n\alpha = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad 2'$$
$$\tan n\alpha = n \tan \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 3'$$
$$\frac{a^2}{b^2} = \frac{n^2 (1 + \tan^2 \alpha)}{n^2 \tan^2 \alpha + 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 4'$$

3′ and 4′ being identical with 3 and 4 above.

"3°. If n be *negative* and equal to $-m$, the equations, (starting from an *apocentral* radius) are

$$x = a\cos\theta + b\cos m\theta$$
$$y = a\sin\theta - b\sin m\theta;$$

which it is clear lead to the same system of equations between a, b, and n as those obtained in the previous sections (1° and 2°).

"4°. If n be *negative* and equal to $-m$, the equations (starting from a *pericentral* radius) are

$$x = a\cos\theta - b\cos n\theta$$
$$y = a\sin\theta + b\sin n\theta;$$

leading, as in the last case, to the original system of equations obtained in sections 1° and 2°.

"Hence we conclude that in all cases the condition of contact is included in the equations

$$\tan n\alpha = n\tan\alpha \quad \ldots\ldots\ldots \quad \text{A}$$

$$\frac{a}{b} = \pm\frac{n.\sqrt{1+\tan^2\alpha}}{\sqrt{n^2\tan^2\alpha + 1}} \quad \ldots\ldots\ldots\ldots \quad \text{B}$$

the n in equations A and B being either the positive or the negative of the n in the proposed curves.

"5°. Since $\tan n\alpha = \dfrac{n\tan\alpha - \dfrac{n.(n-1).(n-2)}{1.2.3}\tan^3\alpha + \&c.}{1 - \dfrac{n(n-1)}{1\ 2}.\tan^3\alpha + \&c.}$

"The equation $\tan na = n\tan\alpha$ becomes

$$\frac{n\tan a - \dfrac{n.(n-1).(n-2)}{1.2.3}\tan^3\alpha + \&c.}{1 - \dfrac{n.(n-1)}{1.2}\tan^2\alpha + \&c.} = n\tan\alpha$$

$$\therefore -\frac{n(n-1)(n-2)}{1.2.3}\tan^3\alpha + \&c.$$

$$= n\tan\alpha\left\{-\frac{n.(n-1)}{1.2}.\tan^2\alpha + \&c.\right\} \quad . \quad . \quad \text{C}$$

"If n be *integral* this equation is finite, and the ratio of a to b can be found by ordinary algebra.

"If n be fractional this equation is transcendental, and the equations between a, b, and n, of the preceding sections can only be regarded as equations of verification.

"E. g.: let $n = 5$, then we get from C

$$-\frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3} \tan^3 \alpha + \tan^5 \alpha$$

$$= 5 \tan \alpha \left\{ -\frac{5 \cdot 4}{1 \cdot 2} \tan^2 \alpha + \frac{5 \cdot 4 \cdot 3 \cdot 2}{1 \cdot 2 \cdot 3 \cdot 4} \tan^4 \alpha \right\}$$

$$\therefore \quad -10 + \tan^2 \alpha = 5 (-10 + 5 \tan^2 \alpha).$$

$$24 \tan^2 \alpha = 40.$$

$$\tan^2 \alpha = \tfrac{5}{3}.$$

$$\therefore \qquad -\frac{a^2}{b^2} = \frac{200}{128} = \frac{100}{64}$$

and $\qquad \therefore \quad \dfrac{a}{b} = \pm \dfrac{5}{4}$

and by what has preceded (sections 3° and 4°) it also follows that if n be equal to -5, the value of $\frac{a}{b}$ is also $\pm \frac{5}{4}$."

LONDON: PRINTED BY
SPOTTISWOODE AND CO., NEW-STREET SQUARE
AND PARLIAMENT STREET